NEW SYLLABUS

PG TRB PHYSICS

EXPERIMENTAL PHYSICS

Professor Academy

PG TRB
PHYSICS

UNIT - X

EXPERIMENTAL PHYSICS

Professor Academy

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Feel free to mail us at **feedback@professoracademy.com**

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UNIT X

SYLLABUS

EXPERIMENTAL PHYSICS

Units and dimension of physical quantities – significant figures. Data interpretation and analysis, precision and accuracy, error analysis, propagation of errors, Least square fitting. Measurement of fundamental constants – e, h, c – Detection of X-rays, gamma rays, Charged particles, neutrons. Ionization chamber – proportional counter – Measurement of e/m ratio – Measurement of Hall voltage, mobility and charge carrier concentration – measurement of resistance and capacitance in series and parallel.

1. UNITS

All the measurable quantities that are used to express the laws of physics are called Physical Quantities. For example: Distance, Mass, Force etc. In our daily life, measuring and comparing the magnitude of different quantities is quite essential. Measurement implies comparison of any unknown physical quantity with a known fixed physical quantity. The known fixed physical quantity is known as unit. In other words, unit is the quantity used as standard for measurement. For example, let the length of the classroom be 10 metre. That means, the length of classroom is compared with the standard quantity of length called metre.

i.e. Physical quantity = value unit

Ex. Length =10 m

C.G.S, F.P.S and M.K.S are the measurement systems that were used for the measurement of physical quantities in earlier times.

C.G.S system: In this system, the unit of length is centimetre, the unit of mass is gram, and the unit of time is second. The CGS system is built on smaller fundamental units, which makes it beneficial in domains such as electromagnetism and optics. Unlike the MKS/SI system, many CGS-derived units have distinct scaling factors, resulting in more difficult conversions. Although the MKS/SI system has completely supplanted the CGS system in most scientific and technical applications, it is still utilized in some fields such as astrophysics and electromagnetism.

F.P.S system: In this system, the unit of length is foot, the unit of mass is pound, and the unit of time is second. This approach is widely utilized in the United States and a few other countries, particularly in engineering and construction. Specific industries, such as aviation and military, still use English units.

M.K.S: In this system, the unit of length is metre, unit of mass is kg, and the unit of time is second. It is a coherent system, which means that derived units are directly based on base units, with no arbitrary conversion factors. It serves as the foundation for the present SI system and is widely utilized in science and most sectors.

S.I System: This system is an improved and extended version of M.K.S system of units. From 1971 till date, the internationally accepted unit system for measurement is **Système International d'units** (SI units).

Important notes:

- In India, the National Physical Laboratory (New Delhi) has the responsibility of maintenance and improvement of physical standards of length, mass, time, etc.
- The 'CGS', 'MKS' and SI units are decimal or metric systems of units and 'FPS' is not a metric system. It is a British system of units.
- In December 1998, the National Aeronautics and Space Administration (NASA), USA, launched the Mars Climate Orbiter to collect data about the Martian climate. Nine months later, on September 23, 1999, the Orbiter disappeared while approaching Mars at an unexpectedly low altitude. An investigation revealed that the orbital calculations were incorrect due to an error in the transfer of information between the spacecraft's team in Colorado and the mission navigation team in California. One team was using the English FPS system of units for calculation, while the other team was using the MKS system of units. This misunderstanding caused a loss of 125 million dollars approximately.

Significance of SI unit:

- Universally accepted for scientific, commercial, and industrial applications.
- It is a logical and decimal-based approach that facilitates conversions.
- SI units are exceedingly accurate and reproducible.
- Encourages worldwide collaboration in science, technology, and commerce.

Classification:

With the development of science & technology, the three fundamental quantities like mass, length & time were not sufficient and hence many other quantities like electric current, heat etc. were introduced. Thus, unit system was modified with addition of four other fundamental quantities and two supplementary quantities. Units are broadly classified into:

- Fundamental (base) units
- Derived units and
- Supplementary units

i) Fundamental (base) units:

There are seven fundamental quantities such as Length, Mass, Time, Electric current, Temperature, Amount of substance and Luminous intensity. From the combinations of these basic quantities, all other physical quantities can be derived. The units corresponding to fundamental quantities are called **fundamental units** and they are listed below.

BASIC QUANTITY	SI UNIT	SYMBOL
Length	Metre	M
Mass	Kilogram	Kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	Mol
Luminous intensity	Candela	Cd

1. Meter (m) - Unit of Length

The unit of length is meter (m). It is used to calculate the separation between two points or the size or extend of items. The meter is defined as the distance that light travels in a vacuum at a time interval of 1/299,792,458 seconds.

2. Kilogram (kg) - Unit of Mass

The unit used for defining mass is kilogram (kg). That is, it is a tool for calculating an object's mass. The kilogram is defined as the mass of a platinum-iridium cylinder, which serves as the worldwide prototype for the kilogram, or as the Planck constant.

The range of masses for different objects:

Object	Order of Mass (kg)
Electron	10^{-30} kg
Proton or Neutron	10^{-27} kg
Uranium atom	10^{-25} kg
Red blood corpuscle	10^{-14} kg
A cell	10^{-10} kg

Object	Order of Mass (kg)
Dust particle	10^{-9} kg
Raindrop	10^{-6} kg
Mosquito	10^{-5} kg
Grape	10^{-3} kg

Frog	10^{-1} kg
Human	10^{2} kg
Car	10^{3} kg
Ship	10^{5} kg
Moon	10^{23} kg

Earth	10^{25} kg
Sun	10^{30} kg
Milky way	10^{41} kg
Observable Universe	10^{55} kg

3. Second (s) – Unit of Time

The unit used for defining time is second (s). It is used to calculate Used to calculate how long events or the time between them last. The second is defined by the frequency of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom. It is **9,192,631,770 periods** of this radiation.

The order of time intervals for different events:

Event	Order of Time Interval (s)
Lifespan of the most unstable particle	10^{-24} s
Time taken by light to cross a distance of nuclear size	10^{-22} s
Period of X-rays	10^{-19} s
Time period of electron in hydrogen atom	10^{-15} s
Period of visible light waves	10^{-15} s
Time taken by visible light to cross through a windowpane	10^{-8} s
Lifetime of an excited state of an atom	10^{-8} s
Period of radio waves	10^{-6} s
Time period of audible sound waves	10^{-3} s
Wink of an eye	10^{-1} s
Travel time of light from Moon to Earth	10^{2} s
Travel time of light from Sun to Earth	10^{2} s
Half-life time of a free neutron	10^{3} s
Time period of a satellite	10^{4} s
Time period of rotation of Earth around its axis (one day)	10^{5} s
Time period of revolution of Earth around the Sun (one year)	10^{7} s
Average life of a human being	10^{9} s
Age of Egyptian pyramids	10^{11} s
Age of Universe	10^{17} s

4. Ampere (A) - Unit of Electric Current

The unit used for defining Electric current is Ampere (A). It is used to measure the flow of electric charge in a conductor. The ampere is defined as the constant current that, if maintained in two straight, parallel conductors of infinite length and negligible cross-section, would produce a force of **2×10^{-7} newtons per meter** of length between the conductors.

5. Kelvin (K) - Unit of Temperature

The unit of temperature is kelvin (K). It is used to measure the thermodynamic temperature, which is the degree of hotness or coldness of an object or system.

The kelvin is the fraction **1/273.16** of the thermodynamic temperature of the triple point of water. Absolute zero (0 K) is the point at which molecular motion ceases.

The primary points for the International Practical Temperature Scale of 1968:

Temperature Point	Temperature (°C)	Temperature (°F)
Triple Point of Equilibrium Hydrogen	-259.34	-434.81
Boiling Point of Equilibrium Hydrogen at 25/76 Normal Pressure	-256.108	-428.99
Normal Boiling Point (1 atm) of Equilibrium Hydrogen	-252.87	-423.17
Normal Boiling Point of Neon	-246.048	-410.89
Triple Point of Oxygen	-218.789	-361.82
Normal Boiling Point of Oxygen	-182.962	-297.33
Triple Point of Water	0.01	32.018
Normal Boiling Point of Water	100.00	212.00
Normal Freezing Point of Zinc	419.58	787.24
Normal Freezing Point of Silver	961.93	1763.47
Normal Freezing Point of Gold	1064.43	1947.97

Conversion Formulas:

- Celsius to Fahrenheit: $F = \frac{9}{5} \times C + 32$
- Fahrenheit to Celsius: $C = \frac{5}{9} \times (F - 32)$
- Celsius to Kelvin: $K = C + 273.15$
- Kelvin to Celsius: $C = K - 273.15$
- Fahrenheit to Kelvin: $K = \frac{5}{9} \times (F - 32) + 273.15$
- Kelvin to Fahrenheit: $F = \frac{9}{5} \times (K - 273.15) + 32$The secondary fixed points for the International Practical Temperature Scale of 1968, listing the specific points and their corresponding temperatures in degrees Celsius:

Temperature Point	Temperature (°C)	Temperature Point	Temperature (°C)
Triple point, normal H2	-259.194	Freezing point, Hg	356.66
Boiling point, normal H2	-252.753	Freezing point, S	444.674
Triple point, Ne	-248.595	Freezing point, Cu-Al eutectic	548.23
Triple point, N2	-210.002	Freezing point, Sb	630.74
Boiling point, N2	-195.802	Freezing point, Al	660.74
Sublimation point, CO2 (normal)	-78.476	Freezing point, Cu	1084.5
Freezing point, Hg	-38.862	Freezing point, Ni	1455
Ice point	0	Freezing point, Co	1494
Triple point, phenoxybenzamine	26.87	Freezing point, Pd	1554
Triple point, Benzoic acid	122.37	Freezing point, Pt	1772
Freezing point, In	156.634	Freezing point, Rh	1963
Freezing point, Bi	271.442	Freezing point, Ir	2447

Freezing point, Cd	321.108	Freezing point, W	3387
Freezing point, Pb	327.502		

6. Mole (mol) - Unit of Amount of Substance

The unit for amount of substance is mole (mol). It is used to measure the amount of substance in terms of the number of particles (atoms, molecules, ions, etc.).

The mole is the amount of substance that contains as many entities (atoms, molecules, etc.) as there are in **12 grams of carbon-12**. This is approximately **6.022×10^{23}** entities.

7. Candela (cd) - Unit of Luminous Intensity

The unit for luminous intensity is candela (cd). It is used to measure the perceived power of light in a specific direction.

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency **540×10^{12} hertz** and that has a radiant intensity of **1/683 watts per steradian** in that direction.

ii) Derived units:

The units that are expressed in terms of fundamental units are called **derived units**. For example, the unit of velocity can be derived by finding its relationship with basic quantities such as length and time.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

Thus, the unit of velocity is determined to be m/s. Some of the examples for derived units are as follows:

Physical Quantity	Formula	Symbol	Notation
Area	Length × Width	A	m^2
Volume	Length × Width × Height	V	m^3
Frequency	1 / Period	F	Hz, s^{-1}
Density	Mass / Volume	P	kg/m^3
Velocity	Displacement / Time	V	m/s
Angular Velocity	Angle / Time	Ω	rad/s
Acceleration	Velocity / Time	A	m/s^2
Angular Acceleration	Angular Velocity / Time	A	rad/s^2
Volumetric Flow Rate	Volume / Time	Q	m^3/s
Force	Mass × Acceleration	F	N ($kg \cdot m/s^2$)
Surface Tension	Force / Length	γ, σ	N/m, J/m^2
Pressure	Force / Area	P	N/m^2, Pa ($kg/m \cdot s^2$)
Dynamic Viscosity	Shear Stress / Velocity Gradient	η, μ	$N \cdot s/m^2$, Pl ($kg/m \cdot s$)
Kinematic Viscosity	Dynamic Viscosity / Density	N	m^2/s
Work, Energy	Force × Distance	W, E	J, $N \cdot m$ ($kg \cdot m^2/s^2$)
Power	Work / Time	P	W (J/s)
Heat Flux Density	Heat Flow / Area	Q	W/m^2
Volumetric Heat Release Rate	Heat Flow / Volume	$\dot{Q}$	W/m^3

Heat-Transfer Coefficient	Heat Flow / (Area × Temperature Difference)	H	$W/m^2 \cdot K$
Specific Enthalpy	Enthalpy / Mass	H	J/kg
Specific Heat Capacity	Heat / (Mass × Temperature)	C	$J/kg \cdot K$
Thermal Conductivity	Heat Flow / (Area × Temperature Gradient)	K	$W/m \cdot K$
Mass Flow Rate	Mass / Time	$\dot{m}$	kg/s
Mass Flux Density	Mass Flow Rate / Area	j_m	$kg/m^2 \cdot s$
Mass-Transfer Coefficient	Mass Flow / Area	B	m/s
Electric Charge	Current × Time	Q	$C (A \cdot s)$
Electromotive Force	Work / Charge	E, emf	$V (kg \cdot m^2/A \cdot s^3)$
Electrical Resistance	Voltage / Current	R	$\Omega (kg \cdot m^2/A^2 \cdot s^3)$
Electrical Conductivity	1 / Resistivity	Σ	$S/m (A^2 \cdot s^3/kg \cdot m^3)$
Electric Capacitance	Charge / Voltage	C	$F (A^2 \cdot s^4/kg \cdot m^2)$
Magnetic Flux	Magnetic Field × Area	Φ	$Wb (kg \cdot m^2/A \cdot s^2)$
Inductance	Magnetic Flux / Current	L	$H (kg \cdot m^2/A^2 \cdot s^2)$
Magnetic Permeability	Inductance / Length	μ	$H/m (kg \cdot m/A^2 \cdot s^2)$
Magnetic Flux Density	Magnetic Flux / Area	B	$T (kg/A \cdot s^2)$
Magnetic Field Strength	Magnetic Force / (Current × Length)	H	A/m
Luminous Flux	Luminous Intensity × Solid Angle	Φv	$lm (cd \cdot sr)$
Luminance	Luminous Flux / Area	L	cd/m^2
Illuminance	Luminous Flux / Area	E	$lx (lm/m^2)$

iii) Supplementary units:

In the International System of Units (SI), the term "supplementary units" referred to a specific set of units that were not classified as fundamental (base) or derived units but were used to describe specific quantities related to geometry or angles. However, the distinction of supplementary units has been removed in the current definition of SI units. As of the 2019 redefinition of the SI system, the concept of supplementary units is no longer formally part of the SI system. The supplementary quantities of plane and solid angle were converted into Derived quantities in 1995 (CGPM)

Nevertheless, it's useful to know the historical context and what these units were used for:

SUPPLEMENTARY QUANTITIES	SI UNIT	SYMBOL
Plane angle	Radian	rad
Solid angle	Steradian	sr

1. Radian (rad)

- Quantity: Plane Angle **(**dθ**)**
- Definition: The radian is the angle subtended at the centre of a circle by an arc whose length (ds) is equal to the radius(r) of the circle. $d\theta = \frac{ds}{r}$

- Relationship to SI Base Units: It is a dimensionless quantity because it is defined as the ratio of two lengths (arc length and radius).

2. Steradian (sr)

- Quantity: Solid Angle
- Definition: The steradian is the unit of solid angle in three-dimensional space. A solid angle **(**$d\Omega$**)** is defined by the area on a sphere's surface divided by the square of the radius of the sphere. One steradian is subtended by a spherical surface area equal to the square of the radius. $d\Omega = \frac{dA}{r^2}$ where dA is the area subtended, and r is the radius.

Rules and Conventions for Writing SI Units and Their Symbols

Naming Units: Units named after scientists are not written with a capital initial letter. Examples: newton, henry, ampere, watt.

Symbols for Units Named After Scientists: Symbols of units named after scientists should be written with an initial capital letter. Examples: N for newton, H for henry, A for ampere, W for watt.

Symbols for Units Not Derived from Proper Nouns: Small letters are used as symbols for units not derived from a proper noun. Examples: m for metre, kg for kilogram.

Punctuation: No full stop or other punctuation marks should be used within or at the end of symbols. Example: 50 m (not 50 m.).

Plural Form: Symbols of units are not expressed in plural form. Example: 10 kg (not 10 kgs).

Temperature: When temperature is expressed in kelvin, the degree sign is omitted. Example: 283 K (not 283° K). When expressed in Celsius, the degree sign should be included. Examples: 100°C (not 100 C), 108°F (not 108 F).

Use of Solidus (/): The solidus (/) is recommended for indicating a division of one unit symbol by another. Not more than one solidus should be used. Examples: ms^{-1} or m/s, $JK^{-1}mol^{-1}$ (not J/K/mol).

Spacing: The number and units should be separated by a space. Example: 15 kg m^{-1} s^{-1} (not 15 $kgms^{-1}$).

Accepted Symbols: Only accepted symbols should be used. Examples: ampere (A) should not be written as amp, second (s) should not be written as sec.

Scientific Notation: Numerical values of physical quantities should be written in scientific notation. Example: the density of mercury should be written as 1.36×10^4 kg m^{-3} (not 13600 kg m^{-3}).

The table you provided lists various units that are retained for general use, even though they are outside the International System of Units (SI).

Name	Symbol	Value in SI Unit
Minute	Min	60 s
Hour	h	60 min = 3600 s
Day	d	24 h = 86400 s
Year	y	365.25 d = 3.156×10^7 s
Degree	°	1° = (π / 180) rad
Litre	L	1 $dm^3 = 10^{-3}$ m^3
Tonne	t	10^3 kg
Carat	ct	200 mg
Curie	Ci	3.7×10^{10} disintegrations per second (dps)

Roentgen	R	2.58×10^{-4} C/kg
Quintal	q	100 kg
Barn	b	100 fm² = 10^{-28} m²
Are	a	1 dam² = 10^2 m²
Hectare	ha	1 hm² = 10^4 m²
Standard Atmospheric Pressure	atm	101325 Pa = 1.013×10^5 Pa

Prefixes for Powers of Ten

In the International System of Units (SI), prefixes are used to denote multiples and submultiples of units. These prefixes represent powers of ten and make it easier to express very large or very small quantities. Here is a list of common SI prefixes for powers of ten:

Multiples of Ten

Factor	Name	Symbol
10^{18}	Exa	E
10^{15}	Peta	P
10^{12}	Tera	T
10^9	Giga	G
10^6	Mega	M
10^3	Kilo	K
10^2	Hector	H
10^1	Deca	Da

Submultiples of Ten

Factor	Name	Symbol
10^{-1}	Deci	d
10^{-2}	Centi	c
10^{-3}	Milli	m
10^{-6}	Micro	μ
10^{-9}	Nano	n
10^{-12}	Pico	p
10^{-15}	Femto	f
10^{-18}	Atto	a

Some Important Ranges and Order of Lengths

Size of Objects and Distances	Length (m)
Distance to the boundary of the observable universe	10^{26}
Distance to the Andromeda galaxy	10^{22}
Size of our galaxy	10^{21}
Distance from Earth to the nearest star (other than the Sun)	10^{16}
Average radius of Pluto's orbit	10^{12}
Distance of the Sun from the Earth	10^{11}
Distance of Moon from the Earth	10^8
Radius of the Earth	10^7
Height of Mount Everest above sea level	10^4
Length of a football field	10^2
Thickness of a paper	10^{-4}
Diameter of a red blood cell	10^{-5}
Wavelength of light	10^{-7}
Length of a typical virus	10^{-8}
Diameter of the hydrogen atom	10^{-10}
Size of an atomic nucleus	10^{-14}
Diameter of a proton	10^{-15}

Cosmic Distances: Distances in the universe range up to 10^{26} meters, like the distance to the boundary of the observable universe.

Astronomical Distances: These include distances within our galaxy and solar system, such as the distance to the Andromeda galaxy (10^{22}meters) and the distance from the Earth to the nearest star (10^{16}meters).

Planetary and Terrestrial Distances: Includes distances within the solar system and on Earth, such as the distance from the Sun to the Earth (10^{11}meters) and the height of Mount Everest (10^{4}meters).

Microscopic Lengths: These include the sizes of biological and atomic structures, such as the diameter of a red blood cell (10^{-5}meters) and the diameter of a hydrogen atom (10^{-10}meters).

Subatomic Lengths: The smallest scales include the size of atomic nuclei (10^{-14}meters) and the diameter of a proton (10^{-15}meters).
Note: Chandrasekhar Limit (CSL) is the largest practical unit of mass. 1 CSL = 1.4 times the mass of the Sun The smallest practical unit of time is Shake. 1 Shake = 10^{-8} s

Some Important practical uses of each of the units:

Unit	Symbol	Value	Practical Use
Fermi	fm	$1\,\text{fm} = 10^{-15}\,\text{m}$	Used to describe the size of atomic nuclei and subatomic particles in nuclear physics.
Angstrom	Å	$1\,\text{Å} = 10^{-10}\,\text{m}$	Commonly used in materials science, crystallography, and the study of atomic structures.
Nanometer	nm	$1\,\text{nm} = 10^{-9}\,\text{m}$	Important in nanotechnology, semiconductor manufacturing, and the study of molecular biology.
Micron	μm	$1\,\mu\text{m} = 10^{-6}\,\text{m}$	Used to measure the size of cells, bacteria, and small particles in microbiology and medical fields.
Light Year	ly	1 light year $= 9.467 \times 10^{15}\,\text{m}$	Used in astronomy to measure vast distances between stars and galaxies.
Astronomical Unit	AU	$1\,\text{AU} = 1.496 \times 10^{11}\,\text{m}$	Used to measure the distance between celestial bodies, especially the Earth and the Sun.
Parsec	pc	1 parsec $= 3.08 \times 10^{16}\,\text{m}$ $= 3.26$ light years	Used in astronomy to measure distances between stars and galaxies, especially in deep space.

Points to remember:

- Units are classified into Fundamental, derived and supplementary units.
- The seven fundamental quantities are Length, Mass, Time, Electric current, Temperature, Amount of substance and Luminous intensity and the units corresponding to fundamental quantities are called fundamental units.
- Plane angle and solid angle form the supplementary quantities and the units corresponding to supplementary quantities are called supplementary units.
- Derived units are expressed in terms of seven fundamental units and two supplementary units.

PRACTICE QUESTIONS

1. Which of the following is a derived unit?

a) Meter b) Kilogram
c) Newton d) Ampere

2. What is the SI unit of electric current?

a) Volt b) Coulomb
c) Ampere d) Ohm

3. Which of the following quantities is dimensionless?

a) Strain b) Velocity
c) Force d) Energy

4. The unit of measurement for magnetic flux density is:

a) Tesla b) Weber
c) Henry d) Gauss

5. Which of the following pairs is incorrectly matched?

a) Luminous intensity - Joule
b) Electric charge - Coulomb
c) Frequency - Hertz
d) Pressure - Pascal

6. The Planck constant has the dimensions of:

a) Energy b) Action
c) Power d) Momentum

7. Which of the following is not a fundamental SI unit?

a) Second b) Kelvin
c) Mole d) Erg

8. The unit of permittivity of free space (ε_0) is:

a) $C^2/N{\cdot}m$ b) $N{\cdot}m^2/C^2$
c) F/m d) $T{\cdot}m/A$

9. In the CGS system, the unit of viscosity is:

a) Poise b) Pascal-second
c) Centipoise d) Newton-second

10. What is the SI unit for inductance?

a) Henry b) Farad
c) Tesla d) Weber

11. The standard kilogram is defined by:

a) A platinum-iridium cylinder kept in France
b) The mass of 1 liter of water at 4°C
c) The mass of a carbon-12 atom
d) A cylinder of silicon-28

12. Which of the following units is used to measure radioactivity?

a) Gray b) Sievert
c) Becquerel d) Rad

13. The unit "farad" is used to measure:

a) Capacitance b) Inductance
c) Magnetic flux d) Electric potential

14. Convert 5 m/s into km/h.

a) 30 km/h b) 14 km/h
c) 46 km/h d) 18 km/hr

15. What is the SI unit of resistance?

a) Ampere b) Volt
c) Coulomb d) Ohm

16. What is the SI unit of permittivity (ε)?

a) Newton/metre b) coulomb/meter
c) Farads/meter d) Joules/meter

17. What is the SI unit of the Surface tension?

a) N/m b) J/m
c) C/m d) N/m^2

18. What is the SI unit of Heat?

a) Kelvin b) Joule
c) calorie d) Erg

Ans: 1-c,2-c,3-a,4-a,5-a,6-b,7-d,8-c,9-a,10-a,11-a,12-c,13-a,14-d,15-d,16-c,17-a,18-b

2. DIMENSIONS OF PHYSICAL QUANTITY

The dimension of a physical quantity is a way of expressing how that quantity is related to the base quantities of the International System of Units (SI), like **length (L)**, **mass (M)**, **time (T)**, etc. It provides an abstract description of the physical nature of the quantity, which allows us to understand how it behaves in terms of fundamental physical concepts. The dimensions of five fundamental physical quantities are listed below. They are denoted using square brackets [].

Fundamental Quantities	Dimension
Length	[L]
Mass	[M]
Time	[T]
Electric current	[A]
Temperature	[K] or [θ]

If a physical quantity Q can be expressed as a product of powers of the fundamental quantities, its dimensional formula can be written in the form:

$$Q=[M^aL^bT^cI^d\theta^eN^fJ^g]$$

Here, a,b,c,d,e,f,g are the powers (exponents) to which each base quantity is raised in the expression for the dimensional formula of the fundamental physical quantities like mass, length, time, electric current, thermodynamic temperature, amount of substance and luminous intensity. But usually, the dimension of any physical quantity is represented by the combination of three fundamental quantities like **[M]**, **[L]** and **[T]**.

The dimension of unknown physical quantity can be determined from the formula of the quantity. For example, if we want to find the dimension of Area, then from the formula Area=length x length, its dimension can be predicted to be $[L^2]$ or $[M^oL^2T^o]$ where the **power** represents the dimension of the quantity. In this case, area has no dependence on mass and time and hence its powers are zero. Some of the derived physical quantities are listed below:

Here is the table with the dimensional formulas added for each physical quantity:

Physical Quantity	Expression	Unit	Dimensional Formula
Area	length × breadth	m^2	$[L^2]$
Volume	area × height	m^3	$[L^3]$
Velocity	displacement / time	$m\ s^{-1}$	$[LT^{-1}]$
Acceleration	velocity / time	$m\ s^{-2}$	$[LT^{-2}]$
Angular Velocity	angular displacement / time	$rad\ s^{-1}$	$[T^{-1}]$
Angular Acceleration	angular velocity / time	$rad\ s^{-2}$	$[T^{-2}]$
Density	mass / volume	$kg\ m^{-3}$	$[ML^{-3}]$
Linear Momentum	mass × velocity	$kg\ m\ s^{-1}$	$[MLT^{-1}]$
Moment of Inertia	mass × $(distance)^2$	$kg\ m^2$	$[ML^2]$
Force	mass × acceleration	$kg\ m\ s^{-2}$ or N	$[MLT^{-2}]$
Pressure	force / area	$N\ m^{-2}$ or Pa	$[ML^{-1}T^{-2}]$
Energy (Work)	force × distance	N m or J	$[ML^2T^{-2}]$
Power	work / time	$J\ s^{-1}$ or watt (W)	$[ML^2T^{-3}]$

Impulse	force × time	N s	$[MLT^{-1}]$
Surface Tension	force / length	N m^{-1}	$[MT^{-2}]$
Moment of Force (Torque)	force × distance	N m	$[ML^2T^{-2}]$
Frequency	Cycles ÷ Time	Hertz	$[T^{-1}]$
Electric Charge	current × time	C	[AT]
Current Density	current / area	A m^{-2}	$[AL^{-2}]$
Magnetic Induction	force / (current × length)	N A^{-1} m^{-1} or tesla	$[MLT^{-2}A^{-1}]$
Force Constant	force / displacement	N m^{-1}	$[MT^{-2}]$
Planck's Constant	energy of photon / frequency	J s	$[ML^2T^{-1}]$
Specific Heat (S)	heat energy / (mass × temperature)	J kg^{-1} K^{-1}	$[ML^2T^{-2}K^{-1}]$
Boltzmann Constant (k)	energy / temperature	J K^{-1}	$[ML^2T^{-2}K^{-1}]$
Voltage	Energy ÷ Charge	Volt	$[ML^2T^{-3}A^{-1}]$
Resistance	Voltage ÷ Current	Ohm	$[ML^2T^{-3}A^{-2}]$
Capacitance	Charge ÷ Voltage	Farad	$[M^{-1}L^{-2}T^4A^2]$
Magnetic Flux	Voltage × Time	Weber	$[ML^2T^{-2}A^{-1}]$
Inductance	Magnetic Flux ÷ Current	Henry	$[ML^2T^{-2}A^{-2}]$

Principle of Homogeneity of Dimensions

Definition: The principle of homogeneity of dimensions states that the dimensions of all the terms in a physical expression should be the same. In other words, all terms in an equation must have the same dimensional formula for the equation to be dimensionally consistent.

Example: Consider the equation for motion: $v^2 = u^2 + 2as$ In this equation:

- v^2 (velocity squared) has dimensions $[L^2T^{-2}]$.
- u^2 (initial velocity squared) also has dimensions $[L^2T^{-2}]$.
- 2as (twice acceleration times displacement) has dimensions $[L^2T^{-2}]$.

Since all terms on both sides of the equation have the same dimensions $[L^2T^{-2}]$, the equation is dimensionally homogeneous. This confirms that the equation is dimensionally consistent.

This principle helps to ensure that physical equations are correct and that operations involving physical quantities are consistent with their units of measurement.

Physical Quantity / Equation	**Expression**	**Dimensional Formula**
Kinematic equation for motion	$v^2 = u^2 + 2as$	$[L^2T^{-2}]$
Displacement	$s = ut + \frac{1}{2}at^2$	[L]
Velocity	$v = \frac{s}{t}$	$[LT^{-1}]$
Acceleration	$a = \frac{v - u}{t}$	$[LT^{-2}]$
Force (Newton's 2nd Law)	$F = ma$	$[MLT^{-2}]$
Work	$W = F \times d$	$[ML^2T^{-2}]$
Energy (Kinetic Energy)	$E_k = \frac{1}{2}mv^2$	$[ML^2T^{-2}]$
Gravitational Potential Energy	$U = mgh$	$[ML^2T^{-2}]$

Power	$P = \frac{W}{t}$	$[ML^2T^{-3}]$
Momentum	$p = mv$	$[MLT^{-1}]$
Impulse	$J = F \times t$	$[MLT^{-1}]$
Work-Energy Theorem	$W = \Delta E$	$[ML^2T^{-2}]$
Gravitational Force (Newton's Law)	$F = G\frac{m_1m_2}{r^2}$	$[M^{-1}L^3T^{-2}]$
Electric Force (Coulomb's Law)	$F = k_e\frac{q_1q_2}{r^2}$	$[M^{-1}L^3T^{-2}A^2]$
Ohm's Law (for Electric Circuit)	$V = IR$	$[ML^2T^{-3}A^{-1}]$
Capacitance (for a Parallel Plate Capacitor)	$C = \frac{\varepsilon A}{d}$	$[M^{-1}L^{-3}T^4A^2]$
Inductance	$L = \frac{N^2}{R}$	$[ML^2T^{-2}A^{-2}]$
Wave Speed (in a string)	$v = \sqrt{\frac{T}{\mu}}$	$[LT^{-1}]$
Planck's Equation (Energy of Photon)	$E = h\nu$	$[ML^2T^{-1}]$

Let's test what we have learnt so far: Shall we?

1. Find the dimensional formula for Self-inductance.

a) $ML^2T^{-2}A^{-2}$ b) $ML^2T^{-2}A$

c) ML^2T^{-2} d) $MLT^{-2}A^{-2}$

(HINT: $e = -L\frac{di}{dt}$ where e is the emf/potential)

Answer: a) $ML^2T^{-2}A^{-2}$

2. Which of the following equation is dimensionally incorrect:

a) v=u+at

b) $s=ut+at^2/2$

c) F=ma+bv (where b is a proportional constant)

d) v=u

Answer: c) F=ma+bv

Solution:

- v=u+at [v]=[u]+[at]: Both the terms in LHS and RHS are $[LT^{-1}]$, so it is dimensionally homogeneous.
- $s=ut+at^2/2$ $[s]=[ut]+[at^2/2]$: Both terms are in [L], so it is dimensionally homogeneous.
- F=ma+bv (where b is a proportional constant)$[F]=[M][LT^{-2}]=[MLT^{-2}]$. If b has dimensions [M], then $[bv]=[MLT^{-1}]$, which is inconsistent. The equation is dimensionally incorrect.

3. Find the dimensions of energy (E) using E=F·L.

a) ML^2T^{-2} b) ML^2T

c) MLT^{-2} d) $ML^2T^{-2}A^{-2}$

Answer: a) ML^2T^{-2}

Solution: $[E]=[F]\cdot[L]=[MLT^{-2}]\cdot[L]=[ML^2T^{-2}]$

4. The time period T of a pendulum depends on: Length l, Gravitational acceleration g. Using dimensional analysis, find the formula.

a) $T\propto\sqrt{\frac{l}{g}}$ b) $T\propto\sqrt{\frac{l}{g}m}$

c) $T\propto\sqrt{lg}$ d) $T\propto\sqrt{\frac{g}{l}}$

Answer: a) $T\propto\sqrt{\frac{l}{g}}$

Solution: Assume $T\propto l^a g^b$ so: $[T]=[l]^a[g]^b$

Substitute dimensions: $[T]=[T]^1,[l]=[L],[g]=[LT^{-2}]$

$[T]=[L]^a[LT^{-2}]^b=[L]^{a+b}[T]^{-2b}$

On equating the powers of L and T: For L: a+b=0

For T: −2b=1 so b=−1/2 and after solving for a, we get a=1/2

$T\propto\sqrt{\frac{l}{g}}$

Limitations of Dimensional analysis

- This method gives no information about the dimensionless constants in the formula like 1, 2, ……..π, e (Euler number), etc.
- This method cannot decide whether the given quantity is a vector or a scalar.
- This method is not suitable to derive relations involving trigonometric, exponential and logarithmic functions.
- It cannot be applied to an equation involving more than three physical quantities.

Applications of Dimensional analysis:

Dimensional analysis: Understanding the dimensions of fundamental quantities is crucial for dimensional analysis, which is used to check the consistency of equations and conversions between units.

Physical laws: The dimensions of fundamental quantities help describe how physical laws are formulated. For example, the equation for force F=ma involves mass and acceleration, which is the rate of change of velocity (L/T), so the dimensions of force are $[M][L][T]^{-2}$.

Unit conversions: Knowing the dimensions of fundamental quantities helps in converting between different units and systems of measurement.

Points to remember:

- Usually, the dimension of any physical quantity is represented by the combination of [M], [L] and [T] and the power of each quantity denotes the dimension of that quantity.
- Dimensional analysis is useful in finding the relationship between physical quantities, to check the accuracy of the formula and to find the units of unknown physical quantities.

PRACTICE QUESTIONS

1.If force (F) is expressed as a function of mass (m), length (L), and time (T), what is its dimensional formula?

a) $[M\ L\ T^{-1}]$ b) $[M\ L\ T^{-2}]$
c) $[M\ L^2\ T^{-2}]$ d) $[M^2\ L\ T^{-2}]$

2.Which of the following physical quantities is dimensionless?

a) Refractive index b) Electric charge
c) Magnetic flux d) Thermal conductivity

3.If the velocity (v) of a particle is given by the equation $v = \sqrt{\frac{2E}{m}}$, where E is energy and m is mass, what are the dimensions of E?

a) $[M\ L^2\ T^{-2}]$ b) $[M\ L\ T^{-2}]$
c) $[M^2\ L^2\ T^{-3}]$ d) $[M\ L^2\ T^{-3}]$

4.The dimensions of Planck's constant (h) are:

a) $[M\ L^2\ T^{-1}]$ b) $[M\ L\ T^{-1}]$
c) $[M^2\ L\ T^{-2}]$ d) $[M\ L^2\ T^{-2}]$

5.Which of the following pairs have the same dimensions?

a) Work and Power
b) Force and Pressure
c) Energy and Work
d) Momentum and Force

6.In the equation $y = a\sin(\omega t + kx)$, where y is displacement, t is time, and x is position, what are the dimensions of ω (angular frequency)?

a) $[T^{-1}]$ b) $[L\ T^{-1}]$
c) $[L^{-1}]$ d) $[M\ T^{-2}]$

7.The Buckingham π theorem is used in dimensional analysis to:

a) Convert units from one system to another
b) Determine the dimensions of a physical quantity
c) Reduce the number of variables in a physical problem
d) Find the exact numerical value of physical quantities

8.The dimensional formula for kinematic viscosity is:

a) $[M\ L^{-1}\ T^{-1}]$ b) $[L^2\ T^{-1}]$
c) $[M\ L\ T^{-2}]$ d) $[L\ T^{-2}]$

9.If the period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length and g is the

acceleration due to gravity, what is the dimensional formula for T?

a) $[L\ T^{-1}]$
b) $[M^0\ L^1\ T^{-2}]$
c) $[T^1]$
d) $[L\ T^2]$

10.Which of the following is a correct statement about dimensional homogeneity?

a) All equations must have the same dimensions on both sides
b) Dimensions can be added or subtracted
c) Dimensionless constants affect dimensional analysis
d) Dimensions are only applicable to fundamental quantities

11.The dimensions of magnetic field intensity (H) are:

a) $[M\ T^{-2}\ A^{-1}]$
b) $[M\ L\ T^{-2}\ A^{-1}]$
c) $[M\ L^{-1}\ T^{-2}\ A^{-1}]$
d) $[L^{-1}\ T^{-1}A]$

12.The ratio of two quantities with the same dimensions is:

a) A scalar
b) A vector
c) A dimensionless quantity
d) A unit quantity

13.Dimensional analysis can be used to derive:

a) Numerical values of constants
b) Functional forms of physical laws
c) Exact solutions to differential equations
d) Empirical formulas for complex phenomena

14.The dimensions of electric field intensity (E) are:

a) $[M\ L\ T^{-3}\ A^{-1}]$
b) $[M\ L^2\ T^{-3}\ A^{-1}]$
c) $[M\ L\ T^{-2}\ A^{-1}]$
d) $[M\ L^2\ T^{-2}\ A^{-1}]$

15.The dimensional formula of thermal resistance is:

a) $[M\ L\ T^{-1}\theta^{-1}]$
b) $[M\ L^2\ T^{-2}\ \theta^{-1}]$
c) $[M\ L^2\ T^{-3}\ \theta^{-1}]$
d) $[M^0\ L^0\ T^0\ \theta^{-1}]$

Ans: 1-b,2-a,3-a,4-a,5-c,6-a,7-c,8-b,9-c,10-a,11-d,12-c,13-b,14-a,15-d.

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3. SIGNIFICANT FIGURES

Significant figures (sig figs) are the digits in a measurement that provide useful information about its precision. These statistics are crucial in scientific computations because they represent the accuracy of the measurements used.
The following rules must be remembered while determining the number of significant figures.

1. **All non-zero digits are significant**:
Any digit from 1 to 9 is considered significant. For example, in the number 1342, all digits are non-zero, so it has four significant figures.

2. **All zeros between two non-zero digits are significant**:
Zeros that appear between non-zero digits are significant. For example, 2008 has four significant figures because the zeros are between 2 and 8.

3. **All zeros to the right of a non-zero digit but to the left of a decimal point are significant**:
Zeros that come after a non-zero digit but before the decimal point are considered significant. For example, 30700. has five significant figures because of the trailing zeros before the decimal.

4. **For numbers without a decimal point, terminal or trailing zeros are not significant**:
When there's no decimal point, trailing zeros are not counted as significant. For example, 30700 has only three significant figures because the zeros are not counted.

5. **For numbers less than 1, zeros to the right of the decimal point but to the left of the first non-zero digit are not significant**:
Leading zeros in decimal numbers are not significant. For example, 0.00345 has three significant figures because the zeros before 3 are not significant.

6. **All zeros to the right of a decimal point and to the right of a non-zero digit are significant**:
In decimal numbers, zeros to the right of the decimal and after a non-zero digit are significant. For example, 40.00 has four significant figures, and 0.030400 has five significant figures.

7. **The number of significant figures does not depend on the system of units used**:
The number of significant figures is the same regardless of the units in which the measurement is expressed. For example, 1.53 cm, 0.0153 m, and 0.0000153 km each have three significant figures.

Notes:
Multiplying or dividing exact numbers: Multiplication or division by exact numbers (like 2 in the formula for circumference, $S = 2\pi r$) does not affect the significant figures, as these are considered to have infinite significant figures.

Power of 10 does not affect significant figures: The exponent in scientific notation does not influence the significant figures. For example, $5.70\text{m}, 5.70 \times 10^2\text{cm}, \text{and } 5.70 \times 10^3\text{mm}$ all have three significant figures.

Rounding off
Rounding off means dropping of unwanted/ insignificant figures. Rounding off significant figures is a crucial process in maintaining the correct precision in your calculations and results. Problem: Round off 3.84 to two significant figures.

Here, there are three significant figures and in order to round off, 4 in 3.84 is considered to be insignificant and is dropped off. However, some of the rules has to be followed while dropping off digits.

If the digit to be dropped is smaller than 5, then the preceding digit should be left unchanged:

- When rounding, if the digit being dropped (after the decimal or in the units place) is less than 5, the preceding digit remains the same.
- **Example i**: 7.32 is rounded off to 7.3 because the digit 2 (to be dropped) is smaller than 5.
- **Example ii**: 8.94 is rounded off to 8.9 because the digit 4 (to be dropped) is smaller than 5.

If the digit to be dropped is greater than 5, then the preceding digit should be increased by 1:

- If the digit being dropped is greater than 5, the preceding digit is increased by 1.
- **Example i**: 17.26 is rounded off to 17.3 because the digit 6 (to be dropped) is greater than 5.
- **Example ii**: 11.89 is rounded off to 11.9 because the digit 9 (to be dropped) is greater than 5.

If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1:

- If the digit to be dropped is 5 and is followed by non-zero digits, the preceding digit should be raised by 1.
- **Example i**: 7.352, when rounded off to the first decimal, becomes 7.4 because the digit 5 (to be dropped) is followed by 2, a non-zero digit.
- **Example ii**: 18.159, when rounded off to the first decimal, becomes 18.2 because the digit 5 (to be dropped) is followed by 1, a non-zero digit.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even:

- If the digit being dropped is 5 (or 5 followed by zeros) and the preceding digit is even, the preceding digit remains unchanged.
- **Example i**: 3.45 is rounded off to 3.4 because the digit 5 (to be dropped) is preceded by an even digit, 4.
- **Example ii**: 8.250 is rounded off to 8.2 because the digit 5 (to be dropped) is preceded by an even digit, 2.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1 if it is odd:

- If the digit being dropped is 5 (or 5 followed by zeros) and the preceding digit is odd, the preceding digit is increased by 1.
- **Example i**: 3.35 is rounded off to 3.4 because the digit 5 (to be dropped) is preceded by an odd digit, 3.
- **Example ii**: 8.350 is rounded off to 8.4 because the digit 5 (to be dropped) is preceded by an odd digit, 3.

Rounding off in arithmetic operations:

The significant figures in the result obtained after multiplication and division should not be more than the significant figures of the original numbers. Example while dividing 4.237 by 2.51, the result should have same significant figures as that of the number with least significant figures (that is 2.51). Otherwise, the result must be round off. Thus, the result would be 1.69. This is the case for multiplication as well.

But in the case of addition and subtraction, the result should have same number of decimal places as that of the original value. Example while adding 22.84, 32.304 and 30.314, the result obtained will be 85.458. The addend 22.84 is correct to two decimal places but the result is corrected to 3 decimal places.

It will be contradictory when the result is more precise than the given value. Thus, 85.458 must be round off to 4 significant figures and the answer becomes 85.46. This is the case for subtraction as well.

Come, Let's Solve this Like a Pro!

1. Add 5.67+12.3+0.004 and express the result with the correct number of significant figures.

a)17.974 b) 17.97
c) 17.9 d) 18.0

Answer: d) 18.0

Solution:

First, we add: 5.67+12.3+0.004=17.974. Since the number with the least number of decimal places is 12.3 (1 decimal place), we round the result to 1 decimal place

2. Express the result of the product 4.56×3.2 with the correct number of significant figures.

a)14.592 b) 14.59
c) 14.5 d) 15

Answer: d) 15

Solution:

4.56×3.2=14.592. Since the number with the least significant figures is 3.2 (2 significant figures), we round the result to 2 significant figures.

Points to remember:

- All non-zero digits are significant.
- All zeros trapped between two non-zero digits are significant.
- The ending zeros are not significant if there are no decimal points.
- The zeros that are placed after the decimal point are significant.
- Zeros before and after decimal point are not significant.
- Number of significant figures should not get changed while changing the units.
- While rounding off, if the insignificant number is less than 5, then it can be dropped.
- While rounding off, if the insignificant number is greater than 5, then preceding digit is increased by 1.
- While rounding off, if the insignificant number is exactly 5, then check the preceding digit. If it is even, then insignificant digit is dropped and if it is odd then preceding digit is increased by 1.
- The significant figures in the result obtained after multiplication and division should not be more than the significant figures of the original numbers.
- In the case of addition and subtraction, the result should have same number of decimal places as that of the original value.

PRACTICE QUESTIONS

1. How many significant figures are there in the number 0.004560?
a) 3 b) 4 c) 5 d) 6

2. Which of the following numbers has 4 significant figures?
a) 0.0405 b) 2.500
c) 3000 d) 123.45

3. In the number 7.030, how many significant figures are there?
a) 2 b) 3 c) 4 d) 5

4. When multiplying 6.38 by 2.0, how many significant figures should the result have?
a) 1 b) 2 c) 3 d) 4

5. Which of the following measurements is correctly rounded to three significant figures?
a) 4.0071 → 4.01 b) 0.00345 → 0.0034
c) 78.95 → 79.0 d) 5001 → 500

6. What is the result of 8.59 + 3.41 rounded to the correct number of significant figures?
a) 11.9 b) 12.0 c) 12 d) 11.90

7. Which of the following correctly expresses 0.000620 in scientific notation with the appropriate number of significant figures?
a) 6.2×10^{-4} b) 6.20×10^{-4}
c) 62×10^{-5} d) 0.62×10^{-3}

8. The number 1500 has how many significant figures?
a) 2 b) 3
c) 4 d) Ambiguous

9. How many significant figures are in the measurement 0.0520 m?
a) 2 b) 3 c) 4 d) 5

10. If you multiply 4.56 by 0.030, how many significant figures should your answer have?
a) 1 b) 2 c) 3 d) 4

11. What is the correct number of significant figures in the sum of 12.11 + 0.22 + 3.1?
a) 2 b) 3 c) 4 d) 5

12. The product of 2.50 and 3.40 should be reported with how many significant figures?
a) 1 b) 2 c) 3 d) 4

13. Which of the following numbers does not have 5 significant figures?
a) 0.003205 b) 12300
c) 500.00 d) 205.30

14. When dividing 56.4 by 1.23, to how many significant figures should the result be rounded?
a) 2 b) 3 c) 4 d) 5

15. What is the number of significant figures in 0.00670?
a) 2 b) 3 c) 4 d) 5

Ans: 1-b,2-b,3-c,4-b,5-c,6-a,7-b,8-d,9-b,10-b,11-b,12-c,13-b,14-b,15-b

4. PRECISION AND ACCURACY

Precision and accuracy are the terms used to describe the quality of measurements or results, but they refer to different aspects:

1. Precision

Precision refers to the degree to which repeated measurements under the same conditions yield the same results. A measurement is **precise** if there is little variation or spread in repeated measurements.

- **High Precision**: If a set of measurements is close to each other but not necessarily close to the true value, the measurements are considered precise.
- **Low Precision**: If the measurements vary significantly from each other, they are considered imprecise.

Example: If you measure the length (originally 4 cm) of an object several times and get values like 5.01 cm, 5.00 cm, and 5.02 cm, the measurements are **precise** because they are very close to each other, even if they are not necessarily the true value.

Note: The formula for precision is ***context-dependent***. The following are some popular precision formulas and interpretations:

Precision in Statistics

Precision is often quantified by the **standard deviation** or **variance** of repeated measurements.

$$\textbf{\textit{Precision}} = \frac{\text{True Value}}{\text{Measured value range}}$$

Where:

Alternatively, precision can be expressed as: ***Precision = 1/Standard Deviation***

Higher precision implies lower standard deviation (values are closer together).

Precision in Information Retrieval

In machine learning or classification tasks, precision evaluates the proportion of true positive predictions out of all positive predictions:

$$\text{Precision} = \frac{\text{Truepositives(TP)}}{\text{True Positives (TP)+False Positives (FP)}}$$

Where:

- **True Positives (TP)** = Correctly predicted positive cases.
- **False Positives (FP)** = Incorrectly predicted positive cases.

Instrument Precision

Precision in instrumentation or measurement systems is typically represented by the **repeatability** of measurements and can be calculated using the coefficient of variation (CV):

$$\text{CV} = \frac{\sigma}{\mu} \times 100$$

Where:

- σ = Standard deviation of the measurements.
- μ = Mean of the measurements.

Ways to Improve Precision of Measurement:

- Use High-Resolution Instruments: Select instruments with finer scales or higher sensitivity to detect small variations.

- Control Environmental Conditions: Maintain stable conditions such as temperature, humidity, and pressure to reduce variability. Eliminate vibrations, electromagnetic interference, or other disturbances.
- Standardize Measurement Procedures: Use consistent methods and techniques during data collection. Follow a strict protocol to avoid variations in handling or measurement setup.
- Reduce Random Errors: Repeat measurements multiple times and average the results. Identify and minimize sources of random variability, such as operator differences or transient conditions.
- Ensure Instrument Stability: Regularly calibrate instruments to avoid drift. Use tools designed for long-term stability and repeatability.
- Use Proper Sampling Techniques: Collect representative and evenly distributed samples. Avoid biased sampling methods that introduce variability.
- Minimize Human Errors: Train operators thoroughly in using instruments and following procedures. Use automated systems where feasible to reduce subjective errors.
- Increase Sample Size: Perform more measurements to minimize the influence of random outliers. Larger datasets often result in more consistent results.
- Optimize Instrument Setup: Use appropriate settings, such as correct range or mode for the measurement. Ensure sensors and probes are properly positioned and secured.
- Use High-Quality Reference Standards: Compare measurements to well-defined and traceable reference standards to maintain consistency.
- Apply Statistical Analysis: Use techniques like variance analysis to identify and address sources of imprecision. Eliminate or account for outliers in datasets.

By focusing on these strategies, one can enhance the repeatability and reliability of your measurements, leading to higher precision.

2. Accuracy

Accuracy refers to how close a measurement is to the **true value** or the accepted reference value. A measurement is **accurate** if it is close to the true value, regardless of how variable the repeated measurements are.

- **High Accuracy**: The measurement is close to the true value.
- **Low Accuracy**: The measurement is far from the true value.

A single measurement can be evaluated for accuracy, or the average of multiple measurements can be compared to the true value. Accuracy can be expressed as the percentage of error or deviation:

$$\text{Accuracy (\%)}=\left(1-\frac{\text{error}}{\text{true value}}\right)\text{x}100$$

$$\text{Error} = \text{Measured value} - \text{True value.}$$

Example: If the true length of an object is known to be 5.00 cm, and you measure it to be 5.01 cm, the measurement is **accurate** because it is close to the true value.

To improve accuracy of measurement:

- Calibrate Instruments: To achieve accurate readings, calibrate equipment on a regular basis with established references.
- Use High-Quality Equipment: Choose instruments with the proper sensitivity and resolution for your measurements.

- Minimize systematic errors: Identify and eliminate sources of bias (such as environmental influences or defects in equipment). Apply corrections for known and unavoidable biases.
- Standardize procedures: To reduce variability, measure using consistent and verified methodologies.
- Control Environmental Factors: Maintain steady conditions (temperature, humidity, and vibration). Reduce extraneous influences, such as electromagnetic interference.
- Train operators: Provide comprehensive training on how to operate equipment correctly. Standardize data gathering processes for all personnel. Validate and cross-check results. Compare the results to known standards or reference methods. Perform cross-validation with multiple instruments or approaches.
- Increase measurement repetition: To eliminate random mistakes, take numerous measurements and average the results.

Relationship Between Precision and Accuracy

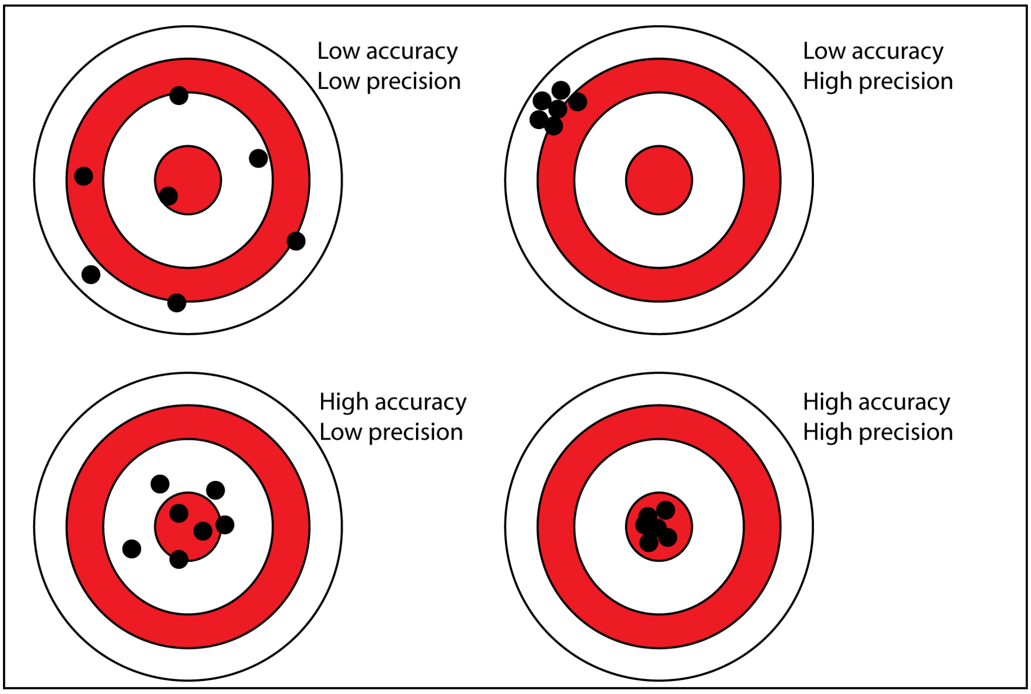
High precision, high accuracy: Measurements are both close to each other and close to the true value. This is ideal.

High precision, low accuracy: Measurements are close to each other but not close to the true value. This indicates a consistent systematic error.

Low precision, high accuracy: Measurements are spread out but on average close to the true value. This may indicate random errors but no systematic bias.

Low precision, low accuracy: Measurements are both spread out and far from the true value. This is undesirable and suggests both random and systematic errors.

The following figure of bullets hitting the target plates explains the concept of accuracy and precision.



Come, Let's Enter Precision Practice Hub

1. Suppose the true value of a length is 5.00 cm, and we measure it several times with the following results:

Trial 1: 5.01 cm, Trial 2: 5.02 cm, Trial 3: 5.00 cm

Identify whether the results are precise or accurate.

a) Measurements are precise and not accurate

b) Measurements are accurate and not precise

c) Measurements are precise and accurate

d) Measurements are neither accurate nor precise

Answer: Option c
Solution:
c) The measurements are **precise** because they are very close to each other. The measurements are also **accurate** because they are close to the true value of 5.00 cm.

2. Given measurements of a liquid's volume:
25.4 mL, 25 mL, 25.5 mL, 25.6 mL
Which measurement has larger deviation?
a) 25.4 mL b) 25 mL
c) 25.5 mL d) 25.6 mL

Answer: b) 25 mL
Solution:
Mean=Sum of measurements/Number of measurements= (25.4+25+25.5+25.6)/4=25.375 mL
Deviation=Measured Value−Mean
- For 25.4; 25.4−25.375=0.025
- For 25: 25−25.375=0.375 (Larger Deviation)
- For 25.5: 25.5−25.375=0.125
- For 25.6: 25.6−25.375=0.225

PRACTICE QUESTIONS

1. Which of the following best describes accuracy?
a) The closeness of a measurement to the true value
b) The consistency of repeated measurements
c) The number of significant figures in a measurement
d) The range of possible values in a measurement

2. Precision is best defined as:
a) The closeness of a measurement to the true value
b) The consistency of repeated measurements
c) The smallest unit of measurement
d) The ability to measure something accurately

3. If a set of measurements are very close to each other but far from the true value, they are:
a) Accurate but not precise
b) Precise but not accurate
c) Both accurate and precise
d) Neither accurate nor precise

4. Which of the following represents high precision and high accuracy?
a) Measurements that are clustered together and close to the true value
b) Measurements that are scattered and far from the true value
c) Measurements that are clustered together but far from the true value
d) Measurements that are scattered but close to the true value

5. An instrument that gives the same reading every time for the same quantity is said to be:
a) Accurate
b) Precise
c) Both accurate and precise
d) Neither accurate nor precise

6. If an archer hits the same spot on a target every time but that spot is not the bullseye, their shots are:
a) Accurate but not precise
b) Precise but not accurate
c) Both accurate and precise
d) Neither accurate nor precise

7. Which of the following is true about systematic errors?
a) They affect the precision of measurements
b) They affect the accuracy of measurements
c) They occur randomly and unpredictably
d) They cannot be corrected by calibration

8. Random errors primarily affect:
a) Accuracy
b) Precision
c) Both accuracy and precision
d) Neither accuracy nor precision

9. An accurate instrument must:
a) Have a high degree of precision
b) Always give the true value of the measured quantity
c) Give measurements that are close to the true value on average
d) Have no systematic errors

10. Which of the following can improve the precision of measurements?
a) Calibrating the instrument
b) Taking multiple measurements and averaging them
c) Using a more accurate instrument
d) Reducing systematic errors

11. If the mean of a large number of measurements is close to the true value, the measurements are considered:
a) Precise but not accurate
b) Accurate but not precise
c) Both accurate and precise
d) Neither accurate nor precise

12. Systematic errors can be minimized by:
a) Increasing the number of measurements
b) Using instruments with higher precision
c) Proper calibration and maintenance of instruments
d) Using the average of multiple measurements

13. A laboratory scale gives a reading of 100.05 g for a standard weight of 100 g each time it is used. The scale is:
a) Accurate but not precise
b) Precise but not accurate
c) Both accurate and precise
d) Neither accurate nor precise

14. Which of the following is an example of a random error?
a) A mis calibrated scale consistently giving readings that are too high
b) Fluctuations in temperature affecting measurement readings
c) A clock that runs slow by 5 minutes every hour
d) A voltmeter that always reads 0.2 V too high

15. An experiment with measurements that have both high accuracy and high precision will result in:
a) Values that are close to the true value but widely scattered
b) Values that are close to each other but far from the true value
c) Values that are close to the true value and closely clustered
d) Values that are widely scattered and far from the true value

Ans: 1-a,2-b,3-b,4-a,5-b,6-b,7-b,8-b,9-c,10-b,11-b,12-c,13-b,14-b,15-c.

5. ERROR ANALYSIS

Introduction to Error Estimation

Science relies on experiments and measurements to confirm or challenge theories and hypotheses. However, measurements alone are not meaningful without addressing the processes involved and their associated uncertainties or errors. In this context, "error" doesn't refer to mistakes but rather to an estimate of the measurement's precision. Estimating errors in experimental results is crucial before making any conclusions.

When reporting results, it is standard practice to include both the measured value and its uncertainty. For instance, if the measured time is reported as (6.5 ± 0.2) seconds, it indicates that the time is 6.5 seconds with an uncertainty of 0.2 seconds, meaning the time could range between 6.3 and 6.7 seconds. The magnitude of both the measured value and its uncertainty depends on the measurement tool and the method used. To demonstrate this, we will define significant figures and uncertainty using an example.

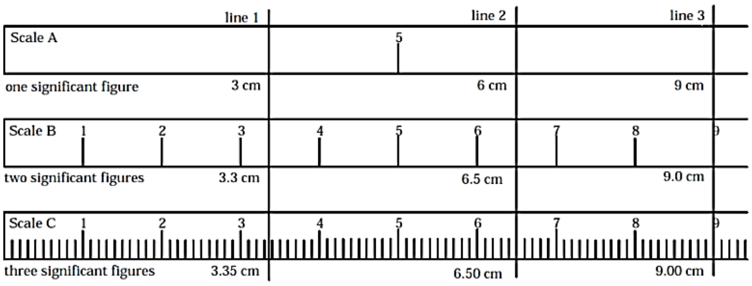


Figure 1. Length measurements of lines 1, 2 and 3 by using rulers with different scales A, B and C.

Imagine three rulers, A, B, and C, each with different scales, are used to measure the lengths of three lines (1, 2, and 3) in Figure 1. These rulers, as different measurement tools, yield varying results even when measuring the same object, due to differences in significant figures and uncertainties. Let's explore why this happens.

Significant Figures and Uncertainty

The numbers derived from measurements are imprecise and subject to error. The precision of a measurement is represented by significant figures, which include all the digits that are directly obtained from the measurement. For example, rulers A, B, and C may measure line 1 as 3, 3.3, and 3.35 cm, respectively, corresponding to 1, 2, and 3 significant figures. More significant figures indicate greater precision, so 3.35 cm (with 3 significant figures) is more accurate than 3 cm (with only 1 significant figure). The number of significant figures is independent of the magnitude of the value. For example, 1234, 12.34, and 1.234 all have four significant figures, despite varying magnitudes.

However, the presence of zeros in measurements requires special attention. There are three types of zeros to consider: leading zeros, trailing zeros, and captive zeros. Leading zeros (e.g., 0.3 or 0.0023) are not significant, as they only serve to position the decimal point. Trailing zeros in a number with a decimal point (e.g., 4500.0) are considered significant, while those without a decimal point (e.g., 4500) are not. Captive zeros, which are between non-zero digits (e.g., 1203.5 or 2.034), are always significant.

To determine significant figures in numbers with zeros, follow these guidelines:

- Leading zeros are not counted.
- Trailing zeros are significant if a decimal point is present.
- Captive zeros are always significant.

For instance, the number 5.294, 0.0003503, and 3.750×10^7 all have four significant figures.

Scientific notation can also simplify the determination of significant figures. For example, writing 13000 as 1.3×10^4 makes it clear that it has 2 significant figures. Similarly, 0.00034 written as 3.4×10^{-4} also has 2 significant figures.

Measurement Examples:

When measuring line 1, using ruler A (with the least precision), the result might be 3 ± 1 cm (1 significant figure), meaning the length is between 2 and 4 cm. Using ruler B (with finer resolution), the result could be 3.3 ± 0.5 cm (2 significant figures), with the length ranging from 2.8 to 3.8 cm. Finally, with ruler C (with the highest precision), the measurement would be 3.35 ± 0.05 cm (3 significant figures), yielding a range of 3.3 to 3.4 cm.

Clearly, the choice of measurement tool affects both the measured value and the uncertainty. More precise instruments yield more significant figures and smaller uncertainties. Therefore, it is beneficial to use tools with higher precision, though they tend to be more expensive.

For lines 2 and 3, the measurements would be consistent with the respective rulers' resolutions. For instance, line 2 might be measured as 6 ± 1 cm (1 significant figure), 6.5 ± 0.5 cm (2 significant figures), or 6.50 ± 0.05 cm (3 significant figures). Similarly, line 3 might be measured as 9 ± 1 cm, 9.0 ± 0.5 cm, or 9.00 ± 0.05 cm.

Consistency Between Significant Figures and Uncertainty

It's important that the number of significant figures in a measurement matches the uncertainty. For example, (21.2 ± 0.2) is valid, but (21.23556 ± 0.2) is not, as the error term has fewer significant figures than the measurement itself. In general, uncertainty should only have one or two digits, and anything more is not acceptable. Therefore, uncertainties like ± 0.3 or ± 0.12 are fine, but ± 0.342 or ± 0.005632 are not.

Mathematical Operations with Significant Figures

When adding or subtracting measurements, the result should be rounded to the least number of decimal places among the numbers involved. For example, (4.5 ± 0.1) cm and (0.3352 ± 0.0002) cm should be summed as 4.8 cm (rounding to 1 decimal place). Similarly, for multiplication and division, the result should have the same number of significant figures as the number with the least significant figures in the calculation.

When performing mixed operations, it's essential to follow the correct order of operations: parentheses, exponents, multiplication and division, and finally, addition and subtraction.

For transcendental functions like trigonometric or logarithmic functions, determining significant figures can be more complicated, but some guidelines are available. The significant figures of a function depend on the number of significant figures in the input value and the magnitude of the result. For example, cos(1.3 rad) = 0.26749… should be reported as 0.27, as 2 significant figures are sufficient to distinguish the change in the value.

Scale Uncertainties in Analog and Digital instruments

Analog Instruments

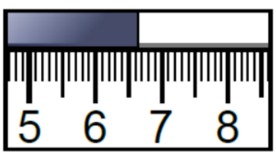


Figure 2. Length measurement by an analog ruler.

Analog instruments like rulers and needle meters rely on visual inspection of the scale markings to estimate measurements. The uncertainty is typically determined by the smallest division on the scale and is often estimated to be half of the smallest scale division. For example, if you measure a length with an analog ruler and get a reading of 6.65 cm, the uncertainty might be ±0.05 cm. If a magnifier is used, the reading could be more precise, such as 6.63 ± 0.04 cm, but it still depends on how carefully the observer reads the scale.

Digital Instruments

Figure 4. Direct current measurement by a digital multimeter displaying a stable reading.

Digital instruments, such as multimeters, display measurements directly as numerical values. These instruments often provide a clearer and more precise way to report uncertainty like in figure 4.

For example, Like in figure 5, if a digital multimeter reads a direct current of 0.320 A, the uncertainty is determined based on the smallest measurable change. In this case, the uncertainty is ±0.005 A, so the reading is reported as (0.320 ± 0.005) A

Figure 5. Direct current measurement by a digital multimeter displaying an unstable reading.

Figure 6. Direct current measurement by a digital multimeter displaying a strongly fluctuating reading.

If the reading fluctuates between 0.32, 0.33, and 0.34 A, the uncertainty would be ±0.015 A, and the result would be reported as (0.33 ± 0.01) A. If the reading fluctuates too much, the uncertainty may be reported as a larger range, such as (0.35 ± 0.05) A.

Estimating Uncertainty in Digital Instruments

In addition to visual inspection of the reading, the specifications of digital instruments can be used to estimate uncertainty.

Function	Range	Resolution	Accuracy
Resistance	200 Ω	0.01 Ω	±(2% + 5 digits)
	2 kΩ	0.1 Ω	±(0.2% + 2 digits)
	20 kΩ	1 Ω	±(0.2% + 2 digits)
	200 kΩ	10 Ω	±(0.2% + 2 digits)
	2000 kΩ	100 Ω	±(0.5% + 2 digits)
	20 MΩ	1 kΩ	±(0.5% + 2 digits)
	200 mV	10 μV	±(0.1% + 4 digits)
DC Voltage	2 V	100 μV	±(0.1% + 4 digits)
	20 V	1 mV	±(0.1% + 4 digits)
	200 V	10 mV	±(0.1% + 4 digits)
	1000 V	100 mV	±(0.15% + 4 digits)
DC Current	2 mA	0.1 μA	±(0.5% + 1 digit)
	20 mA	1 μA	±(0.5% + 1 digit)
	200 mA	10 μA	±(0.5% + 1 digit)
	2000 mA	100 μA	±(0.5% + 1 digit)
	10 A	1 mA	±(0.75% + 3 digits)
AC Voltage (45 Hz – 1 kHz)	200 mV	10 μV	±(0.5% + 20 digits)
	2 V	100 μV	±(0.5% + 20 digits)
	20 V	1 mV	±(0.5% + 20 digits)
	200 V	10 mV	±(0.5% + 20 digits)
	750 V	100 mV	±(1% + 20 digits)

For example, a multimeter might provide a range of uncertainties depending on the function (resistance, DC voltage, etc.) and the measurement range. A resistance measurement on a 200 Ω scale with a resolution of 0.01 Ω might show a reading of 71.49 Ω, and the uncertainty can be calculated as 71.49 × 2% (from the accuracy specification) + 5 dgt (from the resolution), which gives an uncertainty of ±1.48 Ω. Similarly, DC voltage and AC voltage measurements would have uncertainties calculated based on their respective specifications.

Important Points:

- **Analog Instruments:** Scale uncertainty is usually half of the smallest scale division. Accuracy depends on the observer's care and precision of the instrument.
- **Digital Instruments:** Uncertainty is based on the smallest possible change in the displayed reading. This is often more precise and can be determined using the device's specifications.
- **Specifications:** Manufacturers' specifications provide a way to calculate uncertainty by considering the resolution and accuracy for different ranges of measurements.

Sources of Errors

Errors in measurements can arise from three primary sources: the instrument, the method of measurement, and the observed quantity itself. Typically, the largest source of error determines the uncertainty in the data. There are two main types of uncertainty: statistical (random) errors and systematic errors.

Statistical (Random) Errors: These errors arise from unpredictable factors that cause fluctuations in measurements. For instance, if a mechanical stopwatch is aging, it may malfunction, causing the second hand to move either faster or slower at random intervals. This randomness in its movement makes the timing uncertainty unpredictable. When measurements are repeated, the values will vary and exhibit a spread around the average value. This spread is known as random uncertainty. In the following sections, you'll learn that random errors can be estimated and minimized through repeated measurements.

Systematic Errors: These errors cause a consistent bias in one direction, making the measured value either consistently higher or lower than the true value. Systematic errors are often difficult to quantify. For example, if a stopwatch is incorrectly set 5 seconds fast, every measurement will be consistently 5 seconds ahead. This error is not random, and it will persist unless the instrument is corrected. If the stopwatch is lent to someone else without explaining the error, they will unknowingly experience the same 5-second discrepancy. The only way to detect this error is by comparing the stopwatch with another accurate timer.

Another example of a systematic error could occur when using a metal meter stick to measure the length of a table. If the meter stick has contracted due to a change in temperature, it will always measure the table as longer than it actually is, regardless of how carefully the measurement is taken. This systematic error is due to the instrument's material properties being affected by environmental factors (e.g., temperature).

Systematic errors are typically caused by imperfections in the equipment, biased observations, or unaccounted physical effects. Depending on the measurement conditions, an instrument may introduce random errors in some situations and systematic errors in others, or even both at the same time. Thus, it is crucial to identify the source of errors to take appropriate action, such as reducing or estimating the errors.

For instance, in the “Example of Error Analysis” in Appendix A, a free-fall experiment to determine gravitational acceleration (g) uses a stopwatch to measure the time it takes for a ball to fall from a height. The uncertainty in timing is partly due to human reaction time in starting and stopping the stopwatch, which is a random error. Repeated measurements will yield times that are distributed randomly around the true value. On the other hand, air resistance always increases the travel time of the ball, making it a systematic error that consistently affects the results in the same direction.

Statistical (Random) Errors and How to Estimate Them?

Random errors vary with each repetition of a measurement. These fluctuations or instabilities may stem from the observed phenomenon, the measuring instrument, or even the experimenter's actions, and they are beyond our control. Random errors can be minimized by performing repeated measurements. By repeating an experiment many times, we can reduce the impact of random errors and, at the same time, estimate the "true" value of the measurement and its uncertainty.

Statistics provide a powerful method for estimating the magnitude of random errors. When an experiment is repeated several times, the resulting measurements, due to random fluctuations, will form a distribution. This distribution depends on various factors, including the phenomenon being studied and the measurement tools used. Two key distributions often encountered are the **Gaussian** and **Poisson** distributions.

The **Gaussian distribution** applies to measured quantities that have a continuous range of possible values. For example, the length of a table or the gravitational acceleration g are typically described by a Gaussian distribution because values such as 43.232 cm, 43.345 cm, or 43.653 cm for the table's length, or 9.818 m/s^2, 9.806 m/s^2, or 9.823 m/s^2 for gravitational acceleration, are continuous measurements.

On the other hand, the **Poisson distribution** applies to scenarios where only discrete outcomes are possible. For example, counting the number of vehicles passing by in a minute or counting the number of gamma particles emitted from a radioactive source in 30 seconds are examples of processes that follow a Poisson distribution. In these cases, the number of particles or vehicles counted can be 41 or 42, but never a non-integer value like 41.3744. These measurements are discrete and countable.

Gaussian Distribution

Background of Gaussian Distribution

Imagine you are tasked with determining the gravitational acceleration g and you repeat the experiment n times, obtaining values $g_1, g_2, \ldots, g_n$. You can then group the measured values into ranges and plot the frequency of occurrences in a histogram, as shown in Figure 8. This frequency represents the probability of obtaining a particular measured value, normalized by the total number of events.

In this case, the distribution of measurements follows a **Gaussian distribution**, which can be expressed as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean (or expected value) and σ is the standard deviation, as illustrated in Figure 9. The term f(x) dx represents the probability that a measurement will produce a value of x within the interval x to x + dx. Since the sum of all possible measurements must equal 1, we have the equation:

$$\int_{-\infty}^{\infty} f(x)\ dx = 1$$

For continuous random variables, the expected value and variance are defined as:

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, f(x)\, dx\langle x \rangle$$

and

$$\langle(x-\mu)^2\rangle = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\ dx$$

The mean value μ and the standard deviation σ can be calculated from the measurements as:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

and

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\mu)^2$$

Note that the sample standard deviation is used here instead of the population standard deviation because the experiment is considered a finite sample, not the entire population. Therefore, a correction factor is applied by using $n-1$ in the denominator instead of n.

The Gaussian distribution reaches its maximum at x=μ, and this maximum value is:

$$f(\mu) = \frac{1}{\sigma\sqrt{2\pi}}$$

The distribution decreases as x moves away from μ. For values of x one standard deviation away from the mean, i.e., $x = \mu \pm \sigma$, the value of the probability density function drops by a factor of approximately 0.606531. For $x = \mu \pm 2\sigma$, the value drops by a factor of approximately 0.135335. The width of the Gaussian distribution is characterized by the **Full Width at Half Maximum (FWHM)**, which represents the width of the distribution at the point where the value has decreased to half of its maximum. The relationship between the FWHM and the standard deviation is given by:

$$\text{FWHM} = 2\sqrt{2\ln 2}\ \sigma \approx 2.3548$$

To determine the probability that a measured value x falls within a certain range, we can compute the probabilities for specific intervals. For example, the probability that x lies within one standard deviation of the mean is:

$$P(\mu-\sigma \le x \le \mu+\sigma) = 0.682$$

This indicates that there is a 68% chance that a measurement will fall within $\mu \pm \sigma$. Similarly, the probability that x lies within two standard deviations of the mean is:

$$P(\mu-2\sigma \le x \le \mu+2\sigma) = 0.954$$

This means there is a 95% chance that a measurement will fall within $\mu \pm 2\sigma$. Therefore, the standard deviation σ provides a measure of the uncertainty associated with a single measurement.

Estimation of Measured Value and Its Uncertainty

When repeated measurements follow a Gaussian distribution, the "true" value of the measurement is represented by the mean value, and the uncertainty can be determined from the distribution of those measured values. A common question arises: Can we express the result as $x \pm \sigma$ for repeated measurements? The answer is **no**.

Although σ describes the spread of individual measurements, it does not represent the uncertainty of the mean value. To express the uncertainty of the mean correctly, we need to introduce the concept of **standard error**.

If we conduct n repeated measurements at different times, and each measurement gives a value $x_1 \pm \sigma_1, x_2 \pm \sigma_2, \ldots, x_n \pm \sigma_n$, the uncertainty of the mean (also known as the standard error) can be calculated using the following formula:

$$\delta x = \frac{\sigma}{\sqrt{n}}$$

where σ is the standard deviation of the individual measurements. The measured result of repeated measurements is then expressed as:

$$x \pm \delta x$$

It is important to note that the standard error δx is much smaller than the uncertainty from a single measurement due to the $\frac{1}{\sqrt{n}}$ factor.

In practice, even if you only have one set of n measurements, you can still use the standard error formula to calculate the uncertainty. Therefore, the measurement result is presented as:

$$x = x \pm \frac{\sigma}{\sqrt{n}}$$

Example:

Consider a situation where we measure the mass of a sample 30 times, and the measured mass values are listed in Table 1. These values are continuous with three significant figures, as provided by an electronic balance. The uncertainty of each mass value is not specified at this point. We can use the Gaussian approach to estimate the true value and the uncertainty for the entire set of measurements.

1.09	1.01	1.10	1.14	1.16
1.11	1.04	1.16	1.13	1.17
1.14	1.03	1.17	1.09	1.09
1.15	1.06	1.12	1.08	1.20
1.08	1.07	1.14	1.11	1.05
1.06	1.12	1.00	1.10	1.07

Table 1. Measured mass of the sample in kg.

The measured mass values are represented in a histogram (Figure 10), showing the frequency of values in different ranges. Since the error sources are primarily random (due to scale uncertainty, placement of the sample on the balance, etc.), the distribution follows a Gaussian pattern.

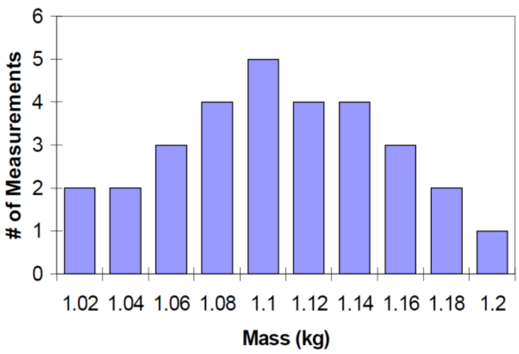


Figure 10. The distribution of the measured mass values.

For the 30 measurements, the mean mass is:

$$m_{mean} = \frac{1}{30}\sum_{i=1}^{30} m_i = 1.10\,\text{kg}$$

From the histogram, it is evident that the data is centered around this mean mass value of 1.10 kg. The standard deviation is calculated as:

$$\sigma_m = \sqrt{\frac{1}{30-1}\sum_{i=1}^{30}(m_i - m_{mean})^2} = 0.05\,\text{kg}$$

The standard error of the measurements is:

$$\delta m = \frac{\sigma_m}{\sqrt{30}} = \frac{0.05}{\sqrt{30}} = 0.01\,\text{kg}$$

Thus, the measured result is presented as:

$$m = (1.10 \pm 0.01)\,\text{kg}$$

This shows the mean value and its associated uncertainty, allowing for a more accurate representation of the measurement.

Summary of Gaussian Errors

Errors generally indicate a range around the measured value where a new measurement is likely to fall. The exact likelihood depends on the statistical distribution of the measurements. Typically, a single measurement has about a 68% chance of being within one standard deviation of the mean. Similarly, the mean has roughly a 68% chance of being within one standard error of the true value. This also means there's a 32% chance that the true value is outside one standard error of the mean. However, this probability decreases significantly as the range around the mean increases. For instance, there's about a 5% chance that the true value is more than two standard errors away from the mean, and less than a 1% chance that it is more than three standard errors away.

This interpretation holds only if the measurements are uncorrelated and free of systematic errors. If instrumental or systematic errors dominate, or if measurements are taken only once or twice, calculating random errors is pointless. Instead, use the scale uncertainty or the best-guess systematic error. When deriving results from multiple measurements, it's crucial to avoid misestimating experimental errors.

It's important to know when to use standard deviation versus standard error. Standard deviation shows the distribution of individual data points around the mean, while standard error indicates the precision of the mean estimate. If you're interested in the spread and variability of data from a single measurement, use standard deviation. For understanding the precision of the true value or comparing differences between means, use standard error.

Key points to remember:

Standard deviation measures how much the values in a dataset differ from each other.
Standard error measures how accurately you know the population mean.
Standard error decreases as sample size increases because larger samples tend to give a mean closer to the true population mean.
Standard deviation does not change predictably with more data; it measures data scattering, which remains consistent regardless of sample size.

Poisson Distribution

Background of Poisson Distribution

Radioactive decay is a process where unstable atoms transform into another element or isotope by emitting photons, electrons, or alpha particles. This decay is an example of a Poisson process, where events are randomly distributed in time, space, or other variables. The detection of particles emitted from a radioactive substance is random and statistically independent, meaning that counting particles over equal time intervals will likely yield different results each time. These counts are subject to statistical fluctuations, and if the experiment is repeated many times, the observed values will follow a distribution based on the number of atoms that can decay and their natural decay rates.

Suppose a sample contains n radioactive nuclei with a known probability of decay p (the decay rate). The probability of recording k counts during a given time interval is given by the Binomial distribution:

$$P(n, p, k) = \frac{n!}{k!\,(n-k)!}p^k(1-p)^{n-k}$$

However, when the number of radioactive nuclei n is much larger than the recorded counts k, and the probability of decay p is small ($n \gg k$ and $p \ll 1$), the Poisson distribution is more suitable. The Poisson distribution is expressed as:

$$P(\mu, k) = \frac{\mu^k e^{-\mu}}{k!},$$

where $\mu = np$. Here, μ is a constant value representing the mean number of counts and is given by the product of n and p.
An example of the Poisson distribution is illustrated in Figure 11.

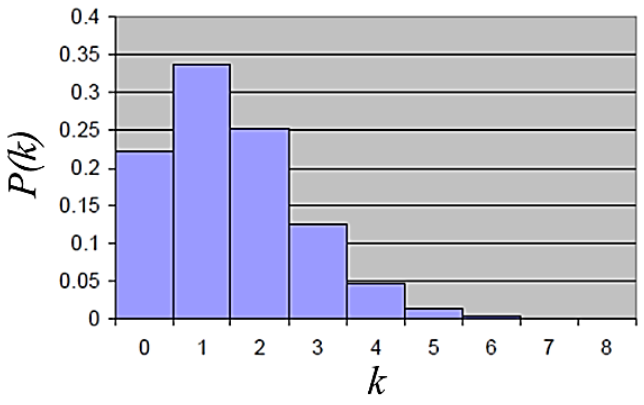


Figure 11. A Poisson distribution when $n >> k$ and $p << 1$.

When both the number of nuclei n and the recorded counts k are large, the Poisson distribution approaches a Gaussian distribution, as shown in Figure 12:

$$P(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-\mu)^2}{2\sigma^2}}$$

where $\sigma^2 = \mu$. The Gaussian distribution is used for continuous random variables, unlike the discrete variables in Poisson and Binomial distributions. This transition to a Gaussian distribution allows the use of more mathematical tools, such as integration, to calculate probabilities.

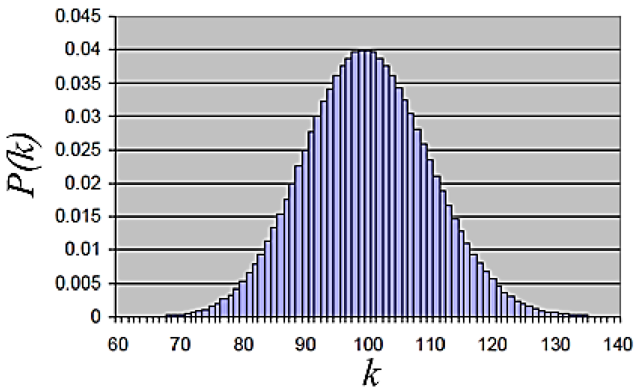


Figure 12. When n and k are large, the distribution is approaching to a Gaussian distribution.

This shift from a Poisson to a Gaussian distribution demonstrates how large datasets enable the application of continuous distribution models and more advanced mathematical methods.

Estimation of Measured Value and Its Uncertainty

Since any observation can yield counts ranging from zero to any positive integer (k = 0, 1, 2, 3, ...), the sum of all probabilities $P(k, \mu)$ for a given μ must equal one:

$$\sum_{k=0}^{\infty} P(k, \mu) = 1.$$

For discrete random variables, the expected value (mean) and variance are defined as $\sum \sum_i x_i P(x_i)$ and $\sum \sum_i (x_i - \mu)^2 P(x_i)$, respectively. For a Poisson distribution, the expected value (mean) μ is given by:
$\mu = \sum_{k=0}^{\infty} k\, P(k, \mu) = \mu$.
The variance, which is equal to σ^2, is:

$$\sigma^2 = \sum_{k=0}^{\infty} (k - \mu)^2 P(k, \mu) = \mu.$$

Thus, the standard deviation σ is:

$$\sigma = \sqrt{\mu}.$$

Assuming most measurements are close to the mean value (i.e., k is near μ), the error of a single measurement can be estimated as $\delta k = \sqrt{k} \approx \sqrt{\mu}$. For example, if 100 particles are counted in 30 seconds, the uncertainty is $\sqrt{100} = 10$, so the resultis 100 ± 10 counts.
An example: counting gamma particles produced from the background over 30 seconds for 1000 trials, with results shown in Table

f	7	34	84	140	176	176	146	104	65	36	18	8	4	1	1	0
k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	≥15

Table 2. The frequency of occurrence f for different counts k.

Here, f is the frequency of occurrence, and k are discrete counts. The sum of all f values gives 1000 measurements, and therefore:

$$P(k) = \frac{f}{1000},$$

$$\sum_{k=0}^{\infty} P(k) = 1.$$

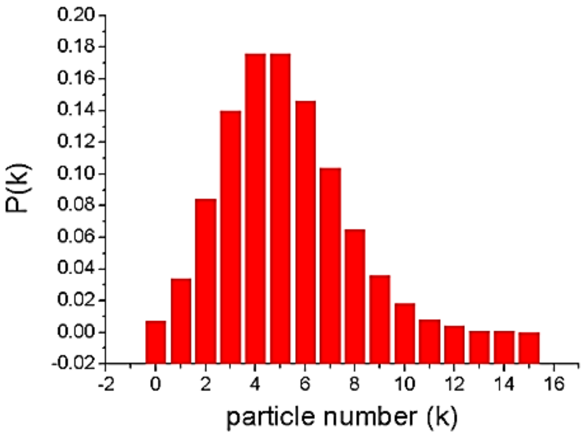


Figure 13. The distribution for different particle count numbers k.

A histogram (Figure 13) shows the distribution. The mean value μ\muμ is calculated as:

$\mu = \sum_k k\,P(k) = 4.997 \approx 5$ counts, close to the peak center. The variance is:

$$\sigma^2 = \sum_i (x_i - \mu)^2 P(x_i) = 5.013 \approx 5,$$

confirming $\sigma^2 = \mu$ and that the data follows a typical Poisson distribution. The standard deviation is:

$$\delta k = \sqrt{\mu} = 2.23 \approx 2 \text{ counts}.$$

Thus, the result can be presented as 5 ± 2 counts. Often, instead of 1000 measurements, a single longer measurement (e.g., 600 seconds) is taken, with the standard deviation given by: $\delta k = \sqrt{k}$.

Systematic Errors

Systematic errors are consistent inaccuracies in measurements caused by factors that have not been properly considered or calibrated. Unlike random errors, systematic errors cannot be reduced by increasing the number of observations. They are more challenging to identify and quantify. Although there is no standard method for calculating systematic errors, they can often be minimized or identified through proper experimental techniques. These errors can be mitigated by using different instruments to cross-check results, having another experimenter repeat the experiment, or improving experimental conditions.

An example of considering systematic errors is the digital multimeter. According to its specifications, the accuracy is (0.025% + 2), where 0.025% is the percentage error relative to the measured value and 2 is the accuracy of the last digit. For a measurement of 2.346 V, the actual voltage is (2.346 ± 0.0006 ± 0.002) V. The first error is the systematic error due to the instrument's gain and offset inaccuracies, while the second is the random error due to scale uncertainty. In precise experiments, these errors should be listed separately because they may contribute differently to the overall experimental errors.

Systematic errors are categorized into two types: instrumentation and environmental. Instrumentation errors can usually be reduced by using higher quality instruments and are easier to estimate. Environmental errors, such as those caused by air friction or Earth's magnetic field, are harder to reduce or estimate and may require redesigning the experiment to mitigate their effects. Computer simulations are often necessary to estimate environmental impacts on experimental results and errors.

Here are some common techniques to minimize systematic errors:

Calibration: Before using any instrument, calibrate it by checking its zero point and taking measurements with a standard reference source.

Comparison with Scale Uncertainties: Compare the error from scale uncertainties with the standard deviation of the measured data. If the standard deviation is larger, it indicates the presence of significant systematic errors.

Independent Experiment Comparison: Compare your results with those from another independent experiment. Discrepancies suggest that at least one experiment has systematic errors.
When conducting experiments, always check the instrument's zeroing before use and compare the results with accepted values. Discuss any potential sources of systematic errors to ensure the accuracy and reliability of your measurements.

Precision vs Accuracy
In scientific experiments, we generally conduct two types. In one type, we aim to verify an existing theory or quantity, such as the gravitational constant. In the other, the theory or quantity is unknown, and our task is to confirm it through research.

For the first type, we focus on accuracy vs. precision. Let's consider an example: Three groups of students attempt to measure the gravitational constant, which is known to be 9.81 m/s^2. After their measurements, all three groups report identical mean values but with different standard errors. We can summarize the results in three cases:

Case 1
$g = 9.75 \pm 0.09\,m/s^2$
The difference between the experimental result and the expected value is 0.06 m/s^2, with a ratio of difference/error = 0.06/0.09, which is less than 1. Therefore, the result is consistent with the expected value.

Case 2
$g = 9.75 \pm 0.01\,m/s^2$
The difference remains 0.06 m/s^2, but the ratio of 0.06/0.01 is much greater than 1. Despite the small error margin, the result is not consistent with the expected value.

Case 3
$g = 9.75 \pm 0.04\,m/s^2$
Here, the difference is still 0.06 m/s^2, but the ratio of 0.06/0.04 is between 1 and 4. This result is inconclusive, suggesting the need for further measurements.

Accuracy refers to how close an experimental result is to the "true" expected value, while precision indicates the consistency of the results, regardless of how close they are to the true value. Thus, Case 2 is more precise than Case 1, likely due to better equipment, though it may not be calibrated correctly. Conversely, Case 1 is more accurate. Ideally, results should be both accurate and precise.

If the ratio of difference to error is consistently greater than 1 across various experiments, the theory or the experimental design may be flawed, necessitating a review of both to explain the discrepancy. Observing such discrepancies can lead to new laws or theories in physics.
In the second type of experiment, where there is no reference to a true value, precision is the main concern. The best result sets a standard until more precise measurements are available. Accuracy is difficult to determine since we don't always know the expected answer, and in scientific research, we rarely know what the answer should be.

In undergraduate physics labs, students typically focus on the first type, studying proven theories or known quantities. Deviations from expected values suggest systematic errors in the experiments, prompting students to identify error sources and improve their methods.
In contrast, research scientists often focus on the second type, where theories or quantities are not yet fully understood.

Propagation of Errors
Often, the physical quantity of interest y is determined as a function f of several measurable quantities x_i (where i=1,2,…,m). Each x_i has a standard error δx_i, and each contributes to the overall error in y.

For instance, in a free-fall experiment, we aim to find g by measuring the distance d and time t independently, then using the equation $g = 2d/t^2$. Suppose $d = (1.095 \pm 0.001)m$ and $t = (0.472 \pm 0.002)$sec. The calculated g would be $g = 2 \times 1.095/0.472^2 = 9.83\, m/s^2$ (with three significant figures). But what about its error?
This section explains how to determine the standard error of a calculated result from the standard errors of the measurements.

The Basic Formula of Error Propagation
When the measurements x_i are uncorrelated, the standard error δy can be estimated using the formula:

$$\delta y = \sqrt{\sum_{i=1}^{m} \left(\frac{\partial f}{\partial x_i} \delta x_i\right)^2}$$

It is important to note that x_i must all be uncorrelated for this equation to be valid. This typically holds true when measurements are taken by different apparatuses, each with independent measurement errors. Do not use this formula if one of the quantities is calculated from the others, as they would not be independent or uncorrelated in such cases.

Some Useful Corollaries
From the basic formula for error propagation, we can derive the results for common functional relationships.
Addition: When quantities are added:

$$y = x_1 + x_2 + x_3 + \cdots$$

The standard error is:

$$\delta y = \sqrt{\delta x_1^2 + \delta x_2^2 + \delta x_3^2 + \ldots}$$

Subtraction: When quantities are subtracted:
$y = x_1 - x_2$
The standard error is:

$$\delta y = \sqrt{\delta x_1^2 + \delta x_2^2}$$

Multiplication: When quantities are multiplied:

$$y = x_1 x_2 x_3 \ldots$$

The standard error is:

$$\frac{\delta y}{y} = \sqrt{\left(\frac{\delta x_1}{x_1}\right)^2 + \left(\frac{\delta x_2}{x_2}\right)^2 + \left(\frac{\delta x_3}{x_3}\right)^2 + \cdots}$$

Division: When quantities are divided: $y = \frac{x_1}{x_2}$

The standard error is:

$$\frac{\delta y}{y} = \sqrt{\left(\frac{\delta x_1}{x_1}\right)^2 + \left(\frac{\delta x_2}{x_2}\right)^2}$$

With a constant: When there is a linear combination of variables with constants $c_1, c_2, \dots$:

$$y = c_1 x_1 + c_2 x_2 + \cdots$$

The standard error is:

$$\delta y = \sqrt{c_1^2 \delta x_1^2 + c_2^2 \delta x_2^2 + \cdots}$$

Power dependence: When quantities have a power dependence:

$$y = x_1^{c_1} x_2^{c_2} \dots$$

The standard error is:

$$\frac{\delta y}{y} = \sqrt{\left(c_1 \frac{\delta x_1}{x_1}\right)^2 + \left(c_2 \frac{\delta x_2}{x_2}\right)^2 + \cdots}$$

By applying these rules of error propagation to the earlier g measurement example, the final result becomes:

$$g = 9.83 \pm 0.08\,\mathrm{m/s^2}$$

Graphical Analysis

In modern physics experiments, most calculations and data analysis are done using computers. However, visualizing the relationships between measurements through simple plots remains a valuable tool. Often, mistakes can be quickly identified by inspecting graphs early in the experimental process, allowing adjustments to be made before too much time is spent. Therefore, understanding basic graphing techniques and methods for extracting information from graphs is important. After that, we will discuss the principle of using computers for curve fitting to determine important parameters.

Conversion to Linear Graphs

Many experiments aim to verify existing theories, for which physical equations are already known. The way we plot the data can simplify the data analysis process. For instance, in a free-fall experiment, to determine the gravitational acceleration g using the equation:

$$d = \frac{1}{2} g t^2$$

where t is the time it takes for the ball to travel a distance d is the independent variable, and t is the dependent variable. Instead of plotting t against d to extract g, it's easier to plot t^2against d. This approach makes it possible to determine g from the slope of the linear plot without needing to fit a curve. More importantly, such a linear plot allows for the immediate identification of any discrepancies before further data analysis.

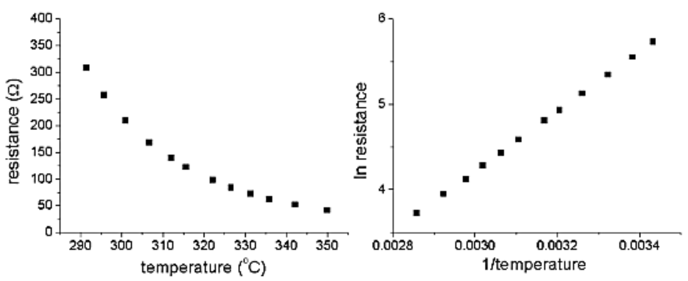


Figure. 14. Plots of resistance against temperature of a semiconductor in two ways.

Another example is the resistance measurement of a semiconductor at different temperatures T. The resistance follows an exponential relationship given by:

$$R = Ae^{bT}$$

where A and b are constants. To determine these parameters, it's easier to plot $ln\,R$ against $1/T$. This will yield a straight line, as shown in Figure 14, from which A and b can be directly extracted from the y-intercept and the slope.

Data Fitting

In physics experiments, it is common to measure a set of n data points(x_i, y_i), where x_i is the independent variable and $y_i$is the dependent variable. The goal is to fit these data points with a smooth function $y = f(x; a, b, c)$, where a, b, and c are the constant parameters to be determined. The function could represent a linear straight line (the simplest case), a higher-order polynomial, or a more complicated form based on theoretical background.

The fitting process allows us to compare experimental data with theoretical predictions and determine the best values for the parameters a, b, and c. For instance, in Figure 15, the black dots represent experimental data points, and the red solid line is the best-fitting curve.

Since curve fitting often involves complex and time-consuming calculations, it is generally not done manually. Many software tools, such as Microsoft Excel, Origin Lab, and MATLAB, offer built-in functions to perform curve fitting. In PHYS1712 lectures, you will be introduced to using Excel's Trendline and Solver for this purpose.

Before applying these tools, it's useful to understand the basic principles of curve fitting. Two common methods for fitting data are **least squares** and **chi-square** fitting. The choice between these methods depends on whether the uncertainty in the data is considered.

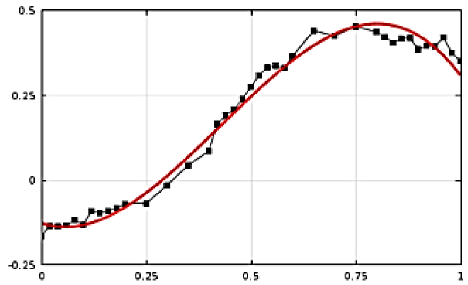


Figure 15. An example of curve fitting in which the black dots are the experimental data and the red solid line is the "best" fitted curve.

Least-Squares Fitting

Basics of Least-Squares Fitting

The definition of the "best" fit is not always straightforward, and sometimes different sets of parameters (a, b, c, ...) can produce curves that appear very close to the data points. Therefore, a criterion is needed to ensure that the data points and the fitting function are as close as possible. The most commonly used method, which is nearly always adopted, is **least squares fitting**. In this method, we minimize the sum of the squares of the differences between the observed y-values (y_i) and the function $y = f(x)$ evaluated at x_i.

Assume we try to fit a function $y = f(x; a, b, c)$to n experimentally determined points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. The goal is to determine the physical parameters a, b, c, etc., for the best fit. To do this, we make the following assumptions:

- $x_i(for\ i = 1,2, \ldots, n)$ are the pre-selected values of the independent variable x and are measured accurately with negligible errors. In other words, there are no uncertainties in x.
- The deviations of $y_i(for\ i = 1,2, \ldots, n)$ from the best curve follow a normal distribution.
- All y_i's are measured with approximately the same level of accuracy.

Under these assumptions, the most likely curve satisfies the **least-squares criterion**. That is, the parameters a, b, c, ... should be chosen to minimize the mean-square deviation S of the y_i's from the curve:

$$S(a, b, c) = \frac{1}{n}\sum_{i=1}^{n}[y_i - f(x_i; a, b, c)]^2$$

To minimize this sum, we set the partial derivatives of S with respect to each parameter equal to zero:

$$\frac{\partial S}{\partial a} = \frac{\partial S}{\partial b} = \frac{\partial S}{\partial c} = 0$$

In most cases, except for very pathological situations, the least-squares criterion is sufficient to determine the parameters a, b, c, ...

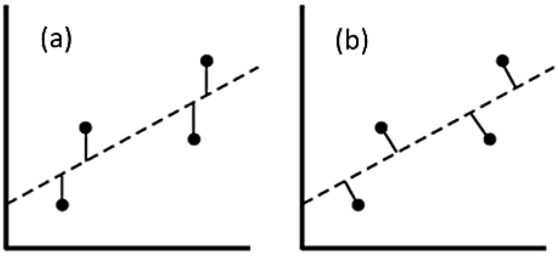


Figure 16. Vertical (a) and perpendicular (b) minimizations.

It's important to note that Equation minimizes the difference in the y-direction only, without accounting for any potential errors in the x-direction. This is shown in **Figure 16(a)**, where the data points are represented by dots, and the dashed line is the fitting function. On the other hand, the minimization can also be done in the direction perpendicular to the fitting function, as shown in **Figure 16(b)**, which involves calculating differences in both the x- and y-directions.

Types of Fitting Functions

The fitting function y=f(x) can take various forms depending on its theoretical background. Common types include:

- **Linear equation**: y=a+bx , where a is the y-intercept and b is the slope.
- **Polynomial equation**: For example, an m[th] order polynomial $y = a_0 + a_1x + a_2x^2 + \cdots + a_mx^m, where\ a_0, a_1, \dots, a_m$ are constants.
- **Exponential and logarithmic forms**: Such as $y = ae^{bx}, y = a\, ln(x)$, or other non-linear forms.

Linear models are relatively simple because the constant parameters are always unique, making them easy to solve directly. However, for nonlinear models, the resulting equations are often nonlinear as well, meaning they may have multiple solutions or no solutions at all. This can make solving for the parameters less straightforward. In such cases, as discussed in it's often advantageous to transform the nonlinear model into a linear one by changing the variables before fitting. Even if such a transformation is not possible, modern mathematical software is powerful enough to handle nonlinear curve fitting.

Least-Squares Fit for a Straight Line

In this section, we apply the least-squares fitting criterion to fit a straight line to a set of data points. The goal is to find the y-intercept a and the slope b of the line, which is described by the equation:

$$\boldsymbol{y = a + bx}$$

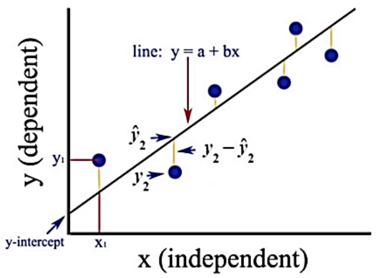


Figure 17. Least-squares fit of the data points (blue dots) to a straight line (black solid line).

Given 6 data points (x_i, y_i) for $i = 1, 2, \dots, 6$, we want to determine the values of a and b that minimize the sum of the squared differences between the observed y_i values and the corresponding y values predicted by the line.

Step-by-Step Derivation

Define the Sum of Squared Differences (S)

The sum of squared differences between the observed and predicted y-values is:

$$S = \sum_{i=1}^{n} \left(y_i - (a + bx_i)\right)^2$$

For n=6, we minimize this sum with respect to a and b.

Minimization with Respect to a

To minimize S, we take the partial derivative of S with respect to a and set it equal to zero:

$$\frac{\partial S}{\partial a} = 0$$

This leads to:

$$\sum_{i=1}^{6}(y_i - a - bx_i)$$

Simplifying:

$$\sum_{i=1}^{6} y_i = 6a + b\sum_{i=1}^{6} x_i$$

Which gives the equation:

$$na + b\sum_{i=1}^{6} x_i = \sum_{i=1}^{6} y_i$$

This simplifies to:

$$na + bX = Y$$

where:

- $X = \sum_{i=1}^{6} x_i$; $Y = \sum_{i=1}^{6} y_i$

Minimization with Respect to b

Next, we take the partial derivative of S with respect to b and set it equal to zero:

$$\frac{\partial S}{\partial b} = 0$$

This leads to:

$$\sum_{i=1}^{6} x_i(y_i - a - bx_i) = 0$$

Simplifying:

$$\sum_{i=1}^{6} x_i y_i = a\sum_{i=1}^{6} x_i + b\sum_{i=1}^{6} x_i^2$$

This gives the equation:

$$aX + bU = Z$$

where:

- $U = \sum_{i=1}^{6} x_i^2$; $Z = \sum_{i=1}^{6} x_i y_i$

Solving the System of Equations

The system of equations we now have is:

$$na + bX = YaX + bU = Z$$

This is a system of two linear equations in the unknowns a and b. The solution can be found by solving this system, which gives:

$$a = \frac{UY - XZ}{nU - X^2} \quad \text{and} \quad b = \frac{nZ - XY}{nU - X^2}$$

These equations can be used to find the best-fit parameters a and b.

Special Cases

Straight Line Through the Origin: If we assume the line passes through the origin (i.e., a=0), the equation simplifies to:

$$y = bx$$

In this case, the slope is given by:

$$b = \frac{Y}{X}$$

Zero Slope: If the slope b = 0, this means the best-fit line is a horizontal line. In this case, the y-intercept a is simply the mean of the $y_i$values:
$a = \frac{Y}{n}$

Check for Consistency

After solving for a and b, we can verify the correctness of the solution by checking if the sum of the residuals (the differences between the observed y_i values and the predicted values $a + bx_i$) is zero. This check is given by the condition:

$$\sum_{i=1}^{n}(y_i - a - bx_i) = 0$$

This provides a simple validation of the computations.

Errors in Least-Squares Fit

In least-squares fitting, it's crucial to understand how errors propagate when determining the parameters a (y-intercept) and b (slope) of the best-fit line. The errors in b and a are related to the uncertainty in the observed data points y_i.

Error Propagation for the Slope b

The error in the slope b, denoted as δb, can be derived using the propagation of error. The formula is:

$$\delta b = \sqrt{\left(\frac{\partial b}{\partial y_i}\delta y_i\right)^2}$$

Given the least-squares solution for b (Equation), we have:

$$\delta b = \frac{\sqrt{\sum_{i=1}^{n}(y_i - \hat{y_i})^2}}{\sqrt{nU - X^2}}$$

Where:

- $\hat{y_i} = a + bx_i$ is the predicted value of $y_i$for each x_i.
- $U = \sum_{i=1}^{n} x_i^2$
- $X = \sum_{i=1}^{n} x_i$

Thus, the standard deviation in b, denoted as δb, depends on the spread of the data points (their deviation from the best-fit line) and the values of X and U.
The equation for the error in b can also be simplified as:

$$\delta b = \frac{\sqrt{n}\,\delta y}{\sqrt{nU - X^2}}$$

Where δy is the standard deviation of y_i values from the best-fit line.

Error Propagation for the Intercept a

Similarly, the error in the intercept a, denoted as δa, can be expressed as:

$$\delta a = \frac{\sqrt{U\,\delta y^2}}{\sqrt{nU - X^2}}$$

This expression indicates that the error in a depends on the spread of the data points as well as the values of X and U.

Standard Deviation of y_i

The standard deviation of the y_i values from the best-fit line can be calculated as:

$$\delta y = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(y_i - a - bx_i)^2}$$

This reflects how much the individual data points deviate from the fitted line. The factor $n-2$ in the denominator accounts for the degrees of freedom, as two parameters (a and b) have been fitted.

PRACTICE QUESTIONS

1. Which of the following is the equation for a Gaussian distribution?

a) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

b) $f(x) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma}\right)$

c) $f(x) = \frac{1}{\sqrt{2\pi\sigma}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

d) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-\frac{(x-\mu)}{2\sigma^2}\right)$

2. In a Gaussian distribution, what percentage of the data falls within one standard deviation from the mean?

a) 68% b) 95% c) 99.7% d) 50%

3. The central limit theorem states that the sum of a large number of small, independent random errors will form a:

a) Uniform distribution b) Binomial distribution

c) Gaussian distribution d) Poisson distribution

4. If the error in a measurement is normally distributed with a mean of 0 and a standard deviation of σ, what is the probability that the error lies with in $\sigma \pm \sigma$?

a) 50% b) 68% c) 95% d) 99%

5. The Poisson distribution is most appropriate for modeling:

a) The distribution of heights in a population

b) The number of defects in a manufactured item

c) The time between arrivals of customers at a store

d) The number of photons detected by a photomultiplier tube

6. In a Poisson distribution, if the average number of occurrences is λ, what is the variance of the distribution?

a) $\sqrt{\lambda}$ b) λ^2 c) λ d) $\frac{1}{\lambda}$

7. Which type of graph is most suitable for visualizing the distribution of a dataset?

a) Scatter plot b) Histogram

c) Line graph d) Bar chart

8. In a linear regression analysis, the slope of the best-fit line represents:

a) The correlation between the variables

b) The intercept on the y-axis

c) The rate of change of the dependent variable with respect to the independent variable

d) The variance of the data

9. Which of the following represents the formula for the uncertainty in the mean value of a set of measurements?

a) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ b) $\sigma_{\bar{x}} = \frac{\sigma^2}{n}$

c) $\sigma_{\bar{x}} = \sqrt{\frac{\sigma}{n}}$ d) $\sigma_{\bar{x}} = \frac{\sigma}{n}$

10. If $z = x \cdot y$, and the errors in x and y are σ_x and σ_y respectively, the propagated error σ_z is given by:

a) $\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial z}{\partial y}\sigma_y\right)^2}$

b) $\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2}$

c) $\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\sigma_x\right) + \left(\frac{\partial z}{\partial y}\sigma_y\right)}$

d) $\sigma_z = \left(\frac{\partial z}{\partial x}\sigma_x\right) + \left(\frac{\partial z}{\partial y}\sigma_y\right)$

11. The standard deviation of a dataset measures:

a) The average value of the dataset

b) The spread or dispersion of the dataset

c) The sum of the squared deviations from the mean

d) The correlation between two variables

12. Given the dataset {2, 4, 4, 4, 5, 5, 7, 9}, what is the standard deviation?

a) 2 b) 3 c) 4 d) 5

Ans: 1-a,2-a,3-c,4-b,5-d,6-c,7-b,8-c,9-a,10-b,11-b,12-a.

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