Proposed Revised Syllabus

Unit I

Analysis:

(1) Elementary set theory, finite, countable and uncountable sets, Archimedean property, supremum, infimum.

(2) Sequences and series, convergence, limsup, liminf.

(3) Bolzano Weierstrass theorem, Heine Borel theorem.

(4) Continuity, uniform continuity, differentiability, mean value theorem.

(5) Sequences and series of functions, uniform convergence.

(6) Riemann sums and Riemann integral, Improper Integrals.

(7) Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral.

(8) Functions of several variables, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.

(9) Metric spaces, compactness, connectedness, Normed linear Spaces, Spaces of continuous functions as examples.

Linear Algebra:

(1) Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.

(2) Algebra of matrices, rank and determinant of matrices, linear equations.

(3) Eigenvalues and eigenvectors, Cayley-Hamilton theorem.

(4) Matrix representation of linear transformations, Change of basis, canonical forms, diagonal forms, triangular forms, jordan forms.

(5) Inner product spaces, orthonormal basis, Symmetric bilinear forms over R, Spectral theorem for normal matrices.

Unit II

Complex Analysis:

(1) Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.

(2) Analytic functions, Cauchy-Riemann equations.

(3) Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem.

(4) Taylor series, Laurent series, calculus of residues.

(5) Mobius transformations.

Algebra and Combinatorics:

(1) Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements.

(2) Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's φ-function.

(3) Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, Class equations, Sylow theorems.

(4) Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.

(5) Polynomial rings and irreducibility criteria.

(6) Fields, finite fields, field extensions, Galois Theory.

Topology:

(1) Basis, dense sets, subspace and product topology, separation axioms, connectedness, compactness and paracompactness.

Unit III

Ordinary Differential Equations:

(1) First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients.

(2) Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method). Sturm's oscillation and separation theorems, Sturm-Liouville eigenvalue problems, boundary value problems and Green's functions.

(3) Systems of linear first order ordinary differential equations, Planar autonomous systems of ordinary differential equations: Stability of stationary points for linear systems with constant coefficients, linearized stability, Lyapunov functions in two variables.

Partial Differential Equations:

(1) Lagrange and Charpit methods, Cauchy problems, method of characteristics for first order partial differential equations.

(2) Second order partial differential equations in two independent variables: classification and canonical forms, Method of separation of variables, Duhamel principle.

(3) Wave equation in one space dimension: Cauchy problem and d'Alembert formula, domain of dependence and range of influence, non-homogeneous equation.

(4) Laplace equation in two variables: Mean value property, maximum principle.

(5) Fourier methods for Laplace, heat, and wave equations.

Numerical Analysis:

(1) Systems of linear equations: Direct methods (Gaussian elimination, LU decomposition, Cholesky factorization), Iterative methods (Gauss-Seidel and Jacobi), and their convergence, Gerschgorin's theorem.

(2) Numerical solutions of nonlinear equations: bisection, regula-falsi, secant, Newton-Raphson methods, fixed point iteration, and their convergence.

(3) Interpolation: Lagrange and Newton forms of interpolating polynomial, Error in polynomial interpolation of a function; Numerical differentiation and error.

(4) Numerical integration: Trapezoidal and Simpson rules, Newton-Cotes integration formulas, composite rules, mathematical errors involved in numerical integration formulae.

(5) Numerical solution of initial value problems for ordinary differential equations: Methods of Euler, Runge-Kutta method of order 2.

Calculus of Variations:

(1) Function spaces: Completeness of C[a, b], C1 [a, b], compact subsets of C[a, b] with supremum norm. Variation of a functional, Euler-Lagrange equation(s), Jacobi necessary condition, Weierstrass-Erdman corner conditions, Jacobi's accessory equation, weak and strong maxima/minima, Weierstrass sufficient conditions, isoperimetric problems, transversality conditions.

Linear Integral Equations:

(1) Linear integral equation of the first and second kind of Fredholm and Volterra type, solutions of integral equations with separable kernels, Laplace transform methods for integral, and integro-differential equations, Characteristic numbers and eigenfunctions, resolvent kernels for Volterra and Fredholm integral equations.

Classical Mechanics:

(1) Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Poisson brackets, Two-dimensional motion of rigid bodies, theory of small oscillations.

Unit IV

Statistics, Probability and Applied Mathematics:

(1) Descriptive statistics, measures of central tendency, measures of dispersion, frequency distribution, histogram, percentiles quartiles kurtosis and skewness, simple, partial and multiple correlation coefficients, scatter plot, exploratory data analysis, box plots, scatter plots, outliers.

(2) Classical and axiomatic definitions of probability, properties of probability function, discrete probability, independent events, Theorem of total probability, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

(3) Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

(4) Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range, bivariate normal distribution and its properties, multinomial distribution.

(5) Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, Neyman-Pearson lemma and its applications in testing simple and composite hypotheses, Unbiased and uniformly most powerful unbiased tests, likelihood ratio tests. Analysis of discrete data and chisquare test of goodness of fit. Large sample tests.

(6) Simple nonparametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.

(7) Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression.

(8) Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques: Principal component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.

(9) Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.

(10) Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2K factorial experiments: confounding and construction.

(11) Hazard function and failure rates, censoring and life testing, series and parallel systems.

(12) Linear programming problem, simplex methods, duality. Elementary queueing and inventory models. Steady-state solutions of Markovian queueing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.