PG TRB PHYSICS

UNIT 1 - MATHEMATICAL PHYSICS

PRACTICE PROBLEMS WITH SOLUTIONS

QUESTION 1

1. Eigenvalues of a Matrix: Calculate the eigenvalues of the matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{4} & \mathbf{2} \\ \mathbf{1} & \mathbf{3} \end{pmatrix}$$

Given:

$$A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$$

Formula:

Characteristic equation (2×2): $\lambda^2 - (\text{Trace})\lambda + \text{Det} = 0$

Trace $= a_{11} + a_{22}$, Det $= a_{11}a_{22} - a_{12}a_{21}$

Check: $\lambda_1 + \lambda_2 = \text{Trace}$, $\lambda_1\lambda_2 = \text{Det}$

Solution:

Trace $= 4 + 3 = 7$

Det $= (4 \times 3) - (2 \times 1) = 12 - 2 = 10$

$\lambda^2 - 7\lambda + 10 = 0$

$(\lambda - 5)(\lambda - 2) = 0$

Answer:

$\lambda = 5, 2$

PRACTICE PROBLEM

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Problem 1

Calculate the eigenvalues of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

Given:

A = $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

Formula:

Characteristic equation (2 × 2): $\lambda^2 - (\text{Trace})\lambda + \text{Det} = 0$

Trace = $a_{11} + a_{22}$, Det = $a_{11}a_{22} - a_{12}a_{21}$

Solution:

Trace = 2 + 2 = 4

Det = (2 × 2) − (1 × 1) = 4 − (1) = 3

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

Check: sum = 4 (trace), product = 3 (det)

Answer:

λ = 3, 1

Problem 2

Calculate the eigenvalues of the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$

Given:

A = $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$

Formula:

Characteristic equation (2 × 2): $\lambda^2 - (\text{Trace})\lambda + \text{Det} = 0$

Trace = $a_{11} + a_{22}$, Det = $a_{11}a_{22} - a_{12}a_{21}$

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Solution:

Trace = 5 + 2 = 7

Det = (5 × 2) − (4 × 1) = 10 − (4) = 6

$\lambda^2 - 7\lambda + 6 = 0$

$(\lambda - 6)(\lambda - 1) = 0$

Check: sum = 7 (trace), product = 6 (det)

Answer:

$\lambda = 6, 1$

Problem 3

Calculate the eigenvalues of the matrix $\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$

Given:

A = $\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$

Formula:

Characteristic equation (2 × 2): $\lambda^2 - (\text{Trace})\lambda + \text{Det} = 0$

Trace = $a_{11} + a_{22}$, Det = $a_{11}a_{22} - a_{12}a_{21}$

Solution:

Trace = 3 + 3 = 6

Det = (3 × 3) − (−1 × −1) = 9 − (1) = 8

$\lambda^2 - 6\lambda + 8 = 0$

$(\lambda - 4)(\lambda - 2) = 0$

Check: sum = 6 (trace), product = 8 (det)

Answer:

$\lambda = 4, 2$

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Problem 4

Calculate the eigenvalues of the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Given:

A = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Formula:

Characteristic equation (2 × 2): $\lambda^2 - (\text{Trace})\lambda + \text{Det} = 0$

Trace = $a_{11} + a_{22}$, Det = $a_{11}a_{22} - a_{12}a_{21}$

Solution:

Trace = 0 + 0 = 0

Det = (0 × 0) − (1 × 1) = 0 − (1) = −1

$\lambda^2 - 0\lambda - 1 = 0 \Rightarrow \lambda^2 = 1$

$(\lambda - 1)(\lambda + 1) = 0$

Check: sum = 0 (trace), product = −1 (det)

Answer:

λ = +1, −1

Problem 5

Calculate the eigenvalues of the matrix $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

Given:

A = $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

Formula:

Characteristic equation (2 × 2): $\lambda^2 - (\text{Trace})\lambda + \text{Det} = 0$

Trace = $a_{11} + a_{22}$, Det = $a_{11}a_{22} - a_{12}a_{21}$

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Solution:

Trace = 4 + 3 = 7

Det = (4 × 3) − (1 × 2) = 12 − (2) = 10

$\lambda^2 - 7\lambda + 10 = 0$

$(\lambda - 5)(\lambda - 2) = 0$

Check: sum = 7 (trace), product = 10 (det)

Answer:

$\lambda = 5, 2$

QUESTION 2

2. Divergence of a Vector Field: Find the divergence of the vector field $A = x^2y\,i + y^2z\,j + z^2x\,k$ at the point (1, 1, 1).

Given:

$A_x = x^2y$, $A_y = y^2z$, $A_z = z^2x$

Point (1, 1, 1)

Formula:

$\nabla \cdot A = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

Solution:

$\partial(x^2y)/\partial x = 2xy$

$\partial(y^2z)/\partial y = 2yz$

$\partial(z^2x)/\partial z = 2zx$

Adding them together:

$\nabla \cdot A = 2xy + 2yz + 2zx$

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At (1, 1, 1): $\nabla \cdot A = 2 + 2 + 2$

Answer:

$\nabla \cdot A = 6$

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Problem 1

Find the divergence of $A = xy\,i + yz\,j + zx\,k$ at the point (1, 2, 3).

Given:

$A_x = xy,\ A_y = yz,\ A_z = zx$

Point (1, 2, 3)

Formula:

$\nabla \cdot A = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

Solution:

$\partial(xy)/\partial x = y$

$\partial(yz)/\partial y = z$

$\partial(zx)/\partial z = x$

$\nabla \cdot A = x + y + z$

At (1, 2, 3): $\nabla \cdot A = 1 + 2 + 3$

Answer:

$\nabla \cdot A = \mathbf{6}$

Problem 2

Find the divergence of $A = x^3\,i + y^3\,j + z^3\,k$ at the point (1, 1, 1).

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Given:

$A_x = x^3,\ A_y = y^3,\ A_z = z^3$

Point (1, 1, 1)

Formula:

$\nabla \cdot A = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

Solution:

$\partial(x^3)/\partial x = 3x^2$

$\partial(y^3)/\partial y = 3y^2$

$\partial(z^3)/\partial z = 3z^2$

$\nabla \cdot A = 3x^2 + 3y^2 + 3z^2$

At (1, 1, 1): $\nabla \cdot A = 3 + 3 + 3$

Answer:

$\nabla \cdot A = \mathbf{9}$

Problem 3

Find the divergence of $A = e^x\, i + \sin y\, j + z^2\, k$ at the point (0, 0, 2).

Given:

$A_x = e^x,\ A_y = \sin y,\ A_z = z^2$

Point (0, 0, 2)

Formula:

$\nabla \cdot A = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

Solution:

$\partial(e^x)/\partial x = e^x$

$\partial(\sin y)/\partial y = \cos y$

$\partial(z^2)/\partial z = 2z$

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$\nabla \cdot A = e^x + \cos y + 2z$

At (0, 0, 2): $\nabla \cdot A = e^0 + \cos 0 + 2(2) = 1 + 1 + 4$

Answer:

$\nabla \cdot A = \mathbf{6}$

Problem 4

Find the divergence of $A = 3x^2\,i - 2y^2\,j + z^2\,k$ at the point $(-1, 2, 1)$.

Given:

$A_x = 3x^2,\ A_y = -2y^2,\ A_z = z^2$

Point $(-1, 2, 1)$

Formula:

$\nabla \cdot A = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

Solution:

$\partial(3x^2)/\partial x = 6x$

$\partial(-2y^2)/\partial y = -4y$

$\partial(z^2)/\partial z = 2z$

$\nabla \cdot A = 6x - 4y + 2z$

At $(-1, 2, 1)$: $\nabla \cdot A = 6(-1) - 4(2) + 2(1) = -6 - 8 + 2$

Answer:

$\nabla \cdot A = \mathbf{-12}$

Problem 5

Find the divergence of $A = x^2yz\,i + xy^2z\,j + xyz^2\,k$ at the point $(1, 1, 1)$.

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Given:

$A_x = x^2yz,\ A_y = xy^2z,\ A_z = xyz^2$

Point (1, 1, 1)

Formula:

$\nabla \cdot A = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

Solution:

$\partial(x^2yz)/\partial x = 2xyz$

$\partial(xy^2z)/\partial y = 2xyz$

$\partial(xyz^2)/\partial z = 2xyz$

$\nabla \cdot A = 6xyz$

At (1, 1, 1): $\nabla \cdot A = 6(1)(1)(1)$

Answer:

$\nabla \cdot A = \mathbf{6}$

QUESTION 3

3. Residue at a Higher-Order Pole: Evaluate the residue of the complex function f(z) = $e^z/(z-2)^2$ at its pole z = 2.

Given:

$f(z) = e^z/(z-2)^2$

Pole z = 2 of order m = 2

Formula:

Pole of order m at z = a:

$\text{Res}(a) = [1/(m-1)!] \lim_{z\to a} d^{m-1}/dz^{m-1} [(z-a)^m f(z)]$

Solution:

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$\text{Res}(2) = (1/1!)\lim_{z\to 2} d/dz\,[(z-2)^2 \cdot e^z/(z-2)^2]$

$\text{Res}(2) = \lim_{z\to 2} d/dz\,[e^z]$

$\text{Res}(2) = \lim_{z\to 2} e^z = e^2$

Answer:

Res = e^2

PRACTICE PROBLEM

Problem 1

Find the residue of $f(z) = e^{2z}/(z-1)^2$ at $z = 1$.

Given:

$f(z) = e^{2z}/(z-1)^2$

Pole $z = 1$ of order $m = 2$

Formula:

Pole of order m at $z = a$:

$\text{Res}(a) = [1/(m-1)!]\lim_{z\to a} d^{m-1}/dz^{m-1}\,[(z-a)^m f(z)]$

Solution:

$(z-1)^2 f(z) = e^{2z}$

$\text{Res} = 1/1! \times \lim_{z\to 1} d/dz\,[e^{2z}] = 2e^{2z}$

$\text{Res} = 2e^{2(1)} = 2e^2$

Answer:

Res = **$2e^2 \approx 14.78$**

Problem 2

Find the residue of $f(z) = z/(z-3)^2$ at $z = 3$.

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Given:

$f(z) = z/(z - 3)^2$

Pole $z = 3$ of order $m = 2$

Formula:

Pole of order m at z = a:

$\text{Res}(a) = [1/(m - 1)!] \lim_{z \to a} d^{m-1}/dz^{m-1} [(z - a)^m f(z)]$

Solution:

$(z - 3)^2 f(z) = z$

$\text{Res} = 1/1! \times \lim_{z \to 3} d/dz [z] = 1$

$\text{Res} = 1$

Answer:

Res = **1**

Problem 3

Find the residue of $f(z) = \sin z/(z - \pi)^2$ at $z = \pi$.

Given:

$f(z) = \sin z/(z - \pi)^2$

Pole $z = \pi$ of order $m = 2$

Formula:

Pole of order m at z = a:

$\text{Res}(a) = [1/(m - 1)!] \lim_{z \to a} d^{m-1}/dz^{m-1} [(z - a)^m f(z)]$

Solution:

$(z - \pi)^2 f(z) = \sin z$

$\text{Res} = 1/1! \times \lim_{z \to \pi} d/dz [\sin z] = \cos z$

$\text{Res} = \cos \pi = -1$

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Answer:

Res = **−1**

Problem 4

Find the residue of $f(z) = e^z/(z + 1)^3$ at $z = -1$.

Given:

$f(z) = e^z/(z + 1)^3$

Pole $z = -1$ of order $m = 3$

Formula:

Pole of order m at z = a:

$\text{Res}(a) = [1/(m - 1)!] \lim_{z \to a} d^{m-1}/dz^{m-1} [(z - a)^m f(z)]$

Solution:

$(z + 1)^3 f(z) = e^z$

$\text{Res} = 1/2! \times \lim_{z \to -1} d^2/dz^2 [e^z] = e^z$

$\text{Res} = (1/2!) e^{-1} = 1/(2e)$

Answer:

Res = **$1/(2e) \approx 0.184$**

Problem 5

Find the residue of $f(z) = z^2/(z - 2)^2$ at $z = 2$.

Given:

$f(z) = z^2/(z - 2)^2$

Pole $z = 2$ of order $m = 2$

Formula:

Pole of order m at z = a:

$\text{Res}(a) = [1/(m - 1)!] \lim_{z \to a} d^{m-1}/dz^{m-1} [(z - a)^m f(z)]$

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Solution:

$(z - 2)^2 f(z) = z^2$

$\text{Res} = 1/1! \times \lim_{z \to 2} d/dz [z^2] = 2z$

$\text{Res} = 2(2) = 4$

Answer:

Res = **4**

QUESTION 4

4. Integral Using the Beta Function: Evaluate the integral $\int [0 \to \pi/2] \sin^5\theta \cos^3\theta \, d\theta$ using the Beta function.

Given:

$p = 5, \ q = 3$

Formula:

$\int [0 \to \pi/2] \sin^p\theta \cos\text{^}q\theta \, d\theta = \Gamma((p+1)/2) \, \Gamma((q+1)/2) / [2 \, \Gamma((p+q+2)/2)]$

$\Gamma(n) = (n - 1)!, \ \Gamma(1/2) = \sqrt{\pi}, \ \Gamma(n + 1) = n\Gamma(n)$

Solution:

$(p + 1)/2 = 3, \ (q + 1)/2 = 2, \ (p + q + 2)/2 = 5$

$I = \Gamma(3) \, \Gamma(2) / [2 \, \Gamma(5)] = (2! \times 1!) / (2 \times 4!)$

$I = 2 / 48 = 1/24$

Answer:

1/24

PRACTICE PROBLEM

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Problem 1

Evaluate $\int [0 \to \pi/2] \sin^3\theta \cos^3\theta \, d\theta$.

Given:

$p = 3,\ q = 3$

Formula:

$\int [0 \to \pi/2] \sin^p\theta \cos^q\theta \, d\theta = \Gamma((p+1)/2)\, \Gamma((q+1)/2) / [2\, \Gamma((p+q+2)/2)]$

$\Gamma(n) = (n-1)!,\ \Gamma(1/2) = \sqrt{\pi},\ \Gamma(n+1) = n\Gamma(n)$

Solution:

$(p+1)/2 = 2,\ (q+1)/2 = 2,\ (p+q+2)/2 = 4$

Numerator: $\Gamma(2)\, \Gamma(2) = 1! \times 1! = 1$

Denominator: $2\, \Gamma(4) = 2 \times 3! = 12$

$I = 1/12$

Answer:

1/12

Problem 2

Evaluate $\int [0 \to \pi/2] \sin^4\theta \cos^2\theta \, d\theta$.

Given:

$p = 4,\ q = 2$

Formula:

$\int [0 \to \pi/2] \sin^p\theta \cos^q\theta \, d\theta = \Gamma((p+1)/2)\, \Gamma((q+1)/2) / [2\, \Gamma((p+q+2)/2)]$

$\Gamma(n) = (n-1)!,\ \Gamma(1/2) = \sqrt{\pi},\ \Gamma(n+1) = n\Gamma(n)$

Solution:

$(p+1)/2 = 5/2,\ (q+1)/2 = 3/2,\ (p+q+2)/2 = 4$

$\Gamma(3/2) = (1/2)\sqrt{\pi},\ \Gamma(5/2) = (3/2)(1/2)\sqrt{\pi} = (3/4)\sqrt{\pi},\ \Gamma(7/2) = (5/2)(3/4)\sqrt{\pi} = (15/8)\sqrt{\pi}$

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Numerator: $\Gamma(5/2)\,\Gamma(3/2) = (3/4)\sqrt{\pi} \times (1/2)\sqrt{\pi} = 3\pi/8$

Denominator: $2\,\Gamma(4) = 2 \times 6 = 12$

$I = (3\pi/8)/12 = 3\pi/96 = \pi/32$

Answer:

$\pi/32$

Problem 3

Evaluate $\int [0 \to \pi/2] \sin^6\theta \cos^0\theta\, d\theta$ ($\cos^0\theta = 1$).

Given:

$p = 6,\ q = 0$

Formula:

$\int [0 \to \pi/2] \sin^p\theta \cos\hat{}q\theta\, d\theta = \Gamma((p+1)/2)\,\Gamma((q+1)/2) / [2\,\Gamma((p+q+2)/2)]$

$\Gamma(n) = (n-1)!,\ \Gamma(1/2) = \sqrt{\pi},\ \Gamma(n+1) = n\Gamma(n)$

Solution:

$(p+1)/2 = 7/2,\ (q+1)/2 = 1/2,\ (p+q+2)/2 = 4$

$\Gamma(3/2) = (1/2)\sqrt{\pi},\ \Gamma(5/2) = (3/2)(1/2)\sqrt{\pi} = (3/4)\sqrt{\pi},\ \Gamma(7/2) = (5/2)(3/4)\sqrt{\pi} = (15/8)\sqrt{\pi}$

Numerator: $\Gamma(7/2)\,\Gamma(1/2) = (15/8)\sqrt{\pi} \times \sqrt{\pi} = 15\pi/8$

Denominator: $2\,\Gamma(4) = 12$

$I = (15\pi/8)/12 = 15\pi/96 = 5\pi/32$

Answer:

$5\pi/32$

Problem 4

Evaluate $\int [0 \to \pi/2] \sin^2\theta \cos^4\theta\, d\theta$.

Given:

$p = 2,\ q = 4$

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Formula:

$\int [0 \to \pi/2] \sin^p\theta \cos^q\theta \, d\theta = \Gamma((p+1)/2)\, \Gamma((q+1)/2) / [2\, \Gamma((p+q+2)/2)]$

$\Gamma(n) = (n-1)!,\ \Gamma(1/2) = \sqrt{\pi},\ \Gamma(n+1) = n\Gamma(n)$

Solution:

$(p+1)/2 = 3/2,\ (q+1)/2 = 5/2,\ (p+q+2)/2 = 4$

$\Gamma(3/2) = (1/2)\sqrt{\pi},\ \Gamma(5/2) = (3/2)(1/2)\sqrt{\pi} = (3/4)\sqrt{\pi},\ \Gamma(7/2) = (5/2)(3/4)\sqrt{\pi} = (15/8)\sqrt{\pi}$

Numerator: $\Gamma(3/2)\, \Gamma(5/2) = (1/2)\sqrt{\pi} \times (3/4)\sqrt{\pi} = 3\pi/8$

Denominator: $2\, \Gamma(4) = 12$

$I = 3\pi/96 = \pi/32$

Answer:

$\pi/32$

Problem 5

Evaluate $\int [0 \to \pi/2] \sin^7\theta \cos^5\theta \, d\theta$.

Given:

$p = 7,\ q = 5$

Formula:

$\int [0 \to \pi/2] \sin^p\theta \cos^q\theta \, d\theta = \Gamma((p+1)/2)\, \Gamma((q+1)/2) / [2\, \Gamma((p+q+2)/2)]$

$\Gamma(n) = (n-1)!,\ \Gamma(1/2) = \sqrt{\pi},\ \Gamma(n+1) = n\Gamma(n)$

Solution:

$(p+1)/2 = 4,\ (q+1)/2 = 3,\ (p+q+2)/2 = 7$

Numerator: $\Gamma(4)\, \Gamma(3) = 3! \times 2! = 12$

Denominator: $2\, \Gamma(7) = 2 \times 6! = 1440$

$I = 12/1440 = 1/120$

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Answer:

1/120

QUESTION 5

5. Poisson Distribution: A radioactive source has an average emission rate of 3 particles per minute. Using the Poisson distribution, find the probability that exactly 2 particles are emitted in a one-minute interval.

Given:

$\lambda = 3,\ x = 2$

$e^{-3} \approx 0.0498$

Formula:

$P(x) = \lambda^x\ e\text{^}(-\lambda) / x!$

Solution:

$P(2) = 3^2\ e^{-3} / 2!$

$P(2) = 9e^{-3} / 2 = 4.5e^{-3}$

$P(2) \approx 4.5 \times 0.0498 \approx 0.224$

Answer:

$P(2) = 4.5e^{-3} \approx 0.224$ (22.4%)

PRACTICE PROBLEM

Problem 1

Average rate $\lambda = 2$. Find the probability of exactly 4 events.

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Given:

$\lambda = 2,\ x = 4$

$e^{-2} \approx 0.1353$

Formula:

$P(x) = \lambda^x\ e\text{^}(-\lambda) / x!$

Substitute:

$P(4) = 2^4\ e^{-2} / 4! = 16e^{-2} / 24$

$P(4) = (2/3)e^{-2}$

$P \approx 0.0902$

Answer:

$P \approx$ **0.0902 (9.0%)**

Problem 2

Average rate $\lambda = 5$. Find the probability of exactly 0 events.

Given:

$\lambda = 5,\ x = 0$

$e^{-5} \approx 0.00674$

Formula:

$P(x) = \lambda^x\ e\text{^}(-\lambda) / x!$

Substitute:

$P(0) = 5^0\ e^{-5} / 0! = 1 \times e^{-5} / 1$

$P(0) = e^{-5}$

$P \approx 0.0067$

Answer:

$P \approx$ **0.0067 (0.67%)**

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Problem 3

Average rate $\lambda = 1$. Find the probability of exactly 3 events.

Given:

$\lambda = 1,\ x = 3$

$e^{-1} \approx 0.3679$

Formula:

$P(x) = \lambda^x\ e\text{^}(-\lambda) / x!$

Substitute:

$P(3) = 1^3\ e^{-1} / 3! = e^{-1} / 6$

$P(3) = 0.3679 / 6$

$P \approx 0.0613$

Answer:

$P \approx$ **0.0613 (6.1%)**

Problem 4

Average rate $\lambda = 4$. Find the probability of exactly 2 events.

Given:

$\lambda = 4,\ x = 2$

$e^{-4} \approx 0.0183$

Formula:

$P(x) = \lambda^x\ e\text{^}(-\lambda) / x!$

Substitute:

$P(2) = 4^2\ e^{-4} / 2! = 16e^{-4} / 2$

$P(2) = 8e^{-4}$

$P \approx 0.1465$

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Answer:

P ≈ **0.1465 (14.7%)**

Problem 5

Average rate λ = 0.5. Find the probability of exactly 1 event.

Given:

λ = 0.5, x = 1

e^(−0.5) ≈ 0.6065

Formula:

$P(x) = \lambda^x e^{(-\lambda)} / x!$

Substitute:

$P(1) = 0.5^1 e^{(-0.5)} / 1!$

$P(1) = 0.5\ e^{(-0.5)}$

P ≈ 0.3033

Answer:

P ≈ **0.3033 (30.3%)**

QUESTION 6

6. Laplace Transform with Shifting: Find the Laplace transform of the function $f(t) = e^{3t}$ cos(2t).

Given:

$f(t) = e^{3t} \cos 2t$

c = 3, a = 2

Formula:

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First shifting theorem: $L\{e^{ct} f(t)\} = F(s - c)$

$L\{\sin at\} = a/(s^2 + a^2)$, $L\{\cos at\} = s/(s^2 + a^2)$

$L\{t^n\} = n!/s^{n+1}$, $L\{\cosh at\} = s/(s^2 - a^2)$

Solution:

$L\{\cos 2t\} = s/(s^2 + 4)$

Replace s by $(s - 3)$: $F(s) = (s - 3)/[(s - 3)^2 + 4]$

$(s - 3)^2 + 4 = s^2 - 6s + 9 + 4 = s^2 - 6s + 13$

Answer: $(s - 3)/(s^2 - 6s + 13)$

PRACTICE PROBLEM

Problem 1

Find the Laplace transform of $f(t) = e^{2t} \sin(3t)$.

Given:

$f(t) = e^{2t} \sin(3t)$

Shift constant $c = 2$

Formula:

First shifting theorem: $L\{e^{ct} f(t)\} = F(s - c)$

$L\{\sin at\} = a/(s^2 + a^2)$, $L\{\cos at\} = s/(s^2 + a^2)$

$L\{t^n\} = n!/s^{n+1}$, $L\{\cosh at\} = s/(s^2 - a^2)$

Solution:

$L\{\sin 3t\} = 3/(s^2 + 9)$

Replace s by $(s - 2)$: $F(s) = 3/[(s - 2)^2 + 9]$

$(s - 2)^2 + 9 = s^2 - 4s + 4 + 9 = s^2 - 4s + 13$

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Answer:

$F(s) = \mathbf{3/(s^2 - 4s + 13)}$

Problem 2

Find the Laplace transform of $f(t) = e^{-t} \cos(4t)$.

Given:

$f(t) = e^{-t} \cos(4t)$

Shift constant $c = -1$

Formula:

First shifting theorem: $L\{e^{ct} f(t)\} = F(s - c)$

$L\{\sin at\} = a/(s^2 + a^2)$, $L\{\cos at\} = s/(s^2 + a^2)$

$L\{t^n\} = n!/s^{n+1}$, $L\{\cosh at\} = s/(s^2 - a^2)$

Solution:

$L\{\cos 4t\} = s/(s^2 + 16)$

Replace s by $(s + 1)$: $F(s) = (s + 1)/[(s + 1)^2 + 16]$

$(s + 1)^2 + 16 = s^2 + 2s + 1 + 16 = s^2 + 2s + 17$

Answer:

$F(s) = \mathbf{(s + 1)/(s^2 + 2s + 17)}$

Problem 3

Find the Laplace transform of $f(t) = e^{4t} t^2$.

Given:

$f(t) = e^{4t} t^2$

Shift constant $c = 4$

Formula:

First shifting theorem: $L\{e^{ct} f(t)\} = F(s - c)$

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$L\{\sin at\} = a/(s^2 + a^2)$, $L\{\cos at\} = s/(s^2 + a^2)$

$L\{t^n\} = n!/s^{n+1}$, $L\{\cosh at\} = s/(s^2 - a^2)$

Solution:

$L\{t^2\} = 2!/s^3 = 2/s^3$

Replace s by $(s - 4)$: $F(s) = 2/(s - 4)^3$

(valid for $s > 4$)

Answer:

$F(s) = \mathbf{2/(s - 4)^3}$

Problem 4

Find the Laplace transform of $f(t) = e^{-3t} \sin(t)$.

Given:

$f(t) = e^{-3t} \sin(t)$

Shift constant $c = -3$

Formula:

First shifting theorem: $L\{e^{ct} f(t)\} = F(s - c)$

$L\{\sin at\} = a/(s^2 + a^2)$, $L\{\cos at\} = s/(s^2 + a^2)$

$L\{t^n\} = n!/s^{n+1}$, $L\{\cosh at\} = s/(s^2 - a^2)$

Solution:

$L\{\sin t\} = 1/(s^2 + 1)$

Replace s by $(s + 3)$: $F(s) = 1/[(s + 3)^2 + 1]$

$(s + 3)^2 + 1 = s^2 + 6s + 9 + 1 = s^2 + 6s + 10$

Answer:

$F(s) = \mathbf{1/(s^2 + 6s + 10)}$

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Problem 5

Find the Laplace transform of $f(t) = e^{5t} \cosh(2t)$.

Given:

$f(t) = e^{5t} \cosh(2t)$

Shift constant $c = 5$

Formula:

First shifting theorem: $L\{e^{ct} f(t)\} = F(s - c)$

$L\{\sin at\} = a/(s^2 + a^2)$, $L\{\cos at\} = s/(s^2 + a^2)$

$L\{t^n\} = n!/s^{n+1}$, $L\{\cosh at\} = s/(s^2 - a^2)$

Solution:

$L\{\cosh 2t\} = s/(s^2 - 4)$

Replace s by $(s - 5)$: $F(s) = (s - 5)/[(s - 5)^2 - 4]$

$(s - 5)^2 - 4 = s^2 - 10s + 25 - 4 = s^2 - 10s + 21 = (s - 3)(s - 7)$

Answer:

$F(s) = \mathbf{(s - 5)/(s^2 - 10s + 21)}$

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