PG TRB PHYSICS

UNIT 8 - NUCLEAR PHYSICS

PRACTICE PROBLEMS WITH SOLUTIONS

QUESTION 1

1. Nuclear Radius: Calculate the nuclear radius of an Aluminium nucleus (^{27}Al). Assume the nuclear radius constant $R_0 = 1.2$ fm.

Given:

A = 27

$R_0 = 1.2$ fm

Formula:

R = R_0 A^(1/3)

Substitute:

R = 1.2 × (27)^(1/3)

(27)^(1/3) = 3

R = 1.2 × 3

R = 3.6 fm

Answer: R = 3.6 fm

PRACTICE PROBLEM

Problem 1

Calculate the nuclear radius of a ^{64}Cu nucleus ($R_0 = 1.2$ fm).

Given:

A = 64

$R_0 = 1.2$ fm

Formula:

R = R_0 A^(1/3)

Substitute:

R = 1.2 × (64)^(1/3)

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(64)^(1/3) = 4

R = 1.2 × 4

R = 4.8 fm

Answer: R = **4.8 fm**

Problem 2

Calculate the nuclear radius of a ^{125}Te nucleus (R_0 = 1.2 fm).

Given:

A = 125

R_0 = 1.2 fm

Formula:

R = R_0 A^(1/3)

Substitute:

R = 1.2 × (125)^(1/3)

(125)^(1/3) = 5

R = 1.2 × 5

R = 6.0 fm

Answer: R = **6.0 fm**

Problem 3

Calculate the nuclear radius of a ^{216}Po nucleus (R_0 = 1.2 fm).

Given:

A = 216

R_0 = 1.2 fm

Formula:

R = R_0 A^(1/3)

Substitute:

R = 1.2 × (216)^(1/3)

(216)^(1/3) = 6

R = 1.2 × 6

R = 7.2 fm

Answer: R = **7.2 fm**

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Problem 4

Calculate the nuclear radius of a ^{8}Be nucleus (R_0 = 1.2 fm).

Given:

A = 8

R_0 = 1.2 fm

Formula:

R = R_0 A^(1/3)

Substitute:

R = 1.2 × (8)^(1/3)

(8)^(1/3) = 2

R = 1.2 × 2

R = 2.4 fm

Answer: R = **2.4 fm**

Problem 5

Calculate the nuclear radius of a ^{343}X (hypothetical) nucleus (R_0 = 1.2 fm).

Given:

A = 343

R_0 = 1.2 fm

Formula:

R = R_0 A^(1/3)

Substitute:

R = 1.2 × (343)^(1/3)

(343)^(1/3) = 7

R = 1.2 × 7

R = 8.4 fm

Answer: R = **8.4 fm**

QUESTION 2

2. Decay Constant: The half-life of a radioactive isotope is 20 days. Calculate its decay constant (λ).

Given:

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$T_{1/2}$ = 20 days

Formula:

$T_{1/2} = 0.693 / \lambda$

$\lambda = 0.693 / T_{1/2}$

Substitute:

$\lambda = 0.693 / 20$

$\lambda = 0.03465\ day^{-1}$

Answer: $\lambda = 0.03465\ day^{-1}$

PRACTICE PROBLEM

Problem 1

The half-life of a radioactive isotope is 5 days. Calculate its decay constant.

Given:

$T_{1/2}$ = 5 days

Formula:

$T_{1/2} = 0.693 / \lambda$

$\lambda = 0.693 / T_{1/2}$

Substitute:

$\lambda = 0.693 / 5$

$\lambda = 0.1386\ day^{-1}$

Answer: λ = 0.1386 day^{-1}

Problem 2

The half-life of a radioactive isotope is 138.6 days. Calculate its decay constant.

Given:

$T_{1/2}$ = 138.6 days

Formula:

$T_{1/2} = 0.693 / \lambda$

$\lambda = 0.693 / T_{1/2}$

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Substitute:

$\lambda = 0.693 / 138.6$

$\lambda = 0.005\ day^{-1}$

Answer: $\lambda =$ **$0.005\ day^{-1}$**

Problem 3

The half-life of a radioactive isotope is 69.3 hours. Calculate its decay constant.

Given:

$T_{1/2} = 69.3$ hours

Formula:

$T_{1/2} = 0.693 / \lambda$

$\lambda = 0.693 / T_{1/2}$

Substitute:

$\lambda = 0.693 / 69.3$

$\lambda = 0.01\ hour^{-1}$

Answer: $\lambda =$ **$0.01\ hour^{-1}$**

Problem 4

The half-life of a radioactive isotope is 10 years. Calculate its decay constant.

Given:

$T_{1/2} = 10$ years

Formula:

$T_{1/2} = 0.693 / \lambda$

$\lambda = 0.693 / T_{1/2}$

Substitute:

$\lambda = 0.693 / 10$

$\lambda = 0.0693\ year^{-1}$

Answer: $\lambda =$ **$0.0693\ year^{-1}$**

Problem 5

The half-life of a radioactive isotope is 2.5 minutes. Calculate its decay constant.

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Given:

$T_{1/2} = 2.5$ minutes

Formula:

$T_{1/2} = 0.693 / \lambda$

$\lambda = 0.693 / T_{1/2}$

Substitute:

$\lambda = 0.693 / 2.5$

$\lambda = 0.2772\ \text{minute}^{-1}$

Answer: λ = 0.2772 minute^{-1}

QUESTION 3

3. Binding Energy: If the mass defect (Δm) of a Helium nucleus (^{4}He) is 0.0304 amu, calculate its total binding energy in MeV. (1 amu = 931.5 MeV)

Given:

$\Delta m = 0.0304$ amu

1 amu = 931.5 MeV

Formula:

$BE = \Delta m \times 931.5$ MeV

Substitute:

$BE = 0.0304 \times 931.5$

BE = 28.3176 MeV

Answer: BE = 28.32 MeV

PRACTICE PROBLEM

Problem 1

If the mass defect of a nucleus is 0.015 amu, calculate its binding energy in MeV.

Given:

$\Delta m = 0.015$ amu

1 amu = 931.5 MeV

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Formula:

$BE = \Delta m \times 931.5 \text{ MeV}$

Substitute:

$BE = 0.015 \times 931.5$

$BE = 13.9725 \text{ MeV}$

Answer: BE = **13.97 MeV**

Problem 2

If the mass defect of a nucleus is 0.120 amu, calculate its binding energy in MeV.

Given:

$\Delta m = 0.120 \text{ amu}$

$1 \text{ amu} = 931.5 \text{ MeV}$

Formula:

$BE = \Delta m \times 931.5 \text{ MeV}$

Substitute:

$BE = 0.120 \times 931.5$

$BE = 111.78 \text{ MeV}$

Answer: BE = **111.78 MeV**

Problem 3

If the mass defect of a nucleus is 0.085 amu, calculate its binding energy in MeV.

Given:

$\Delta m = 0.085 \text{ amu}$

$1 \text{ amu} = 931.5 \text{ MeV}$

Formula:

$BE = \Delta m \times 931.5 \text{ MeV}$

Substitute:

$BE = 0.085 \times 931.5$

$BE = 79.1775 \text{ MeV}$

Answer: BE = **79.18 MeV**

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Problem 4

If the mass defect of a nucleus is 0.500 amu, calculate its binding energy in MeV.

Given:

$\Delta m = 0.500$ amu

1 amu = 931.5 MeV

Formula:

$BE = \Delta m \times 931.5$ MeV

Substitute:

$BE = 0.500 \times 931.5$

$BE = 465.75$ MeV

Answer: BE = **465.75 MeV**

Problem 5

If the mass defect of a nucleus is 0.215 amu, calculate its binding energy in MeV.

Given:

$\Delta m = 0.215$ amu

1 amu = 931.5 MeV

Formula:

$BE = \Delta m \times 931.5$ MeV

Substitute:

$BE = 0.215 \times 931.5$

$BE = 200.2725$ MeV

Answer: BE = **200.27 MeV**

QUESTION 4

4. Fission Energy: A single fission of a Uranium-235 nucleus releases 200 MeV of energy. Calculate the total energy released (in MeV) when 5×10^{10} nuclei undergo fission.

Given:

$N = 5 \times 10^{10}$ nuclei

$E_{single} = 200$ MeV

Formula:

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$E_{total} = N \times E_{single}$

Substitute:

$E_{total} = (5 \times 10^{10}) \times 200$

$E_{total} = 1000 \times 10^{10}$

$E_{total} = 1 \times 10^{13}$ MeV

Answer: $E_{total} = 1 \times 10^{13}$ MeV

PRACTICE PROBLEM

Problem 1

If 1 fission of U-235 releases 200 MeV, find the energy released by 3×10^{8} nuclei.

Given:

$N = 3 \times 10^{8}$ nuclei

$E_{single} = 200$ MeV

Formula:

$E_{total} = N \times E_{single}$

Substitute:

$E_{total} = (3 \times 10^{8}) \times 200$

$E_{total} = 600 \times 10^{8}$

$E_{total} = 6.00 \times 10^{10}$ MeV

Answer: $E_{total} =$ **6.00×10^{10} MeV**

Problem 2

If 1 fission of U-235 releases 200 MeV, find the energy released by 4.5×10^{12} nuclei.

Given:

$N = 4.5 \times 10^{12}$ nuclei

$E_{single} = 200$ MeV

Formula:

$E_{total} = N \times E_{single}$

Substitute:

$E_{total} = (4.5 \times 10^{12}) \times 200$

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$E_{total} = 900 \times 10^{12}$

$E_{total} = 9.00 \times 10^{14}$ MeV

Answer: $E_{total} = \mathbf{9.00 \times 10^{14}}$ **MeV**

Problem 3

If 1 fission of Pu-239 releases 210 MeV, find the energy released by 2×10^{15} nuclei.

Given:

$N = 2 \times 10^{15}$ nuclei

$E_{single} = 210$ MeV

Formula:

$E_{total} = N \times E_{single}$

Substitute:

$E_{total} = (2 \times 10^{15}) \times 210$

$E_{total} = 420 \times 10^{15}$

$E_{total} = 4.20 \times 10^{17}$ MeV

Answer: $E_{total} = \mathbf{4.20 \times 10^{17}}$ **MeV**

Problem 4

If 1 fission of U-235 releases 200 MeV, find the energy released by 8×10^{5} nuclei.

Given:

$N = 8 \times 10^{5}$ nuclei

$E_{single} = 200$ MeV

Formula:

$E_{total} = N \times E_{single}$

Substitute:

$E_{total} = (8 \times 10^{5}) \times 200$

$E_{total} = 1600 \times 10^{5}$

$E_{total} = 1.60 \times 10^{8}$ MeV

Answer: $E_{total} = \mathbf{1.60 \times 10^{8}}$ **MeV**

Problem 5

If 1 fission of U-233 releases 195 MeV, find the energy released by 6×10^{10} nuclei.

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Given:

$N = 6 \times 10^{10}$ nuclei

$E_{single} = 195$ MeV

Formula:

$E_{total} = N \times E_{single}$

Substitute:

$E_{total} = (6 \times 10^{10}) \times 195$

$E_{total} = 1170 \times 10^{10}$

$E_{total} = 1.17 \times 10^{13}$ MeV

Answer: $E_{total} =$ **1.17×10^{13} MeV**

QUESTION 5

5. Gell-Mann–Nishijima Formula: An elementary particle has an isospin projection $I_3 = +1/2$, baryon number B = 1 and strangeness S = 0. Find the electric charge (Q) of the particle.

Given:

$I_3 = +1/2$

$B = 1$

$S = 0$

Formula:

$Q = I_3 + (B + S) / 2$

Substitute:

$Q = 1/2 + (1 + 0) / 2$

$Q = 1/2 + 1/2$

$Q = +1$

Answer: Q = +1 (Proton)

PRACTICE PROBLEM

Problem 1

Find Q for a particle with $I_3 = -1/2$, B = 1, and S = 0 (Neutron).

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Given:

$I_3 = -1/2$

$B = 1$

$S = 0$

Formula:

$Q = I_3 + (B + S) / 2$

Substitute:

$Q = (-1/2) + (1 + 0) / 2$

$Q = (-1/2) + (+1/2)$

$Q = 0$

Answer: $Q =$ **0 (Neutron)**

Problem 2

Find Q for a particle with $I_3 = 0$, $B = 1$, and $S = -1$ (Λ^0 hyperon).

Given:

$I_3 = 0$

$B = 1$

$S = -1$

Formula:

$Q = I_3 + (B + S) / 2$

Substitute:

$Q = (0) + (1 + (-1)) / 2$

$Q = (0) + (0)$

$Q = 0$

Answer: $Q =$ **0 (Λ^0 hyperon)**

Problem 3

Find Q for a particle with $I_3 = +1$, $B = 0$, and $S = 0$ (π^+ meson).

Given:

$I_3 = +1$

$B = 0$

$S = 0$

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Formula:

$Q = I_3 + (B + S) / 2$

Substitute:

$Q = (+1) + (0 + 0) / 2$

$Q = (+1) + (0)$

$Q = +1$

Answer: $Q = +1$ **(π^+ meson)**

Problem 4

Find Q for a particle with $I_3 = -1/2$, $B = 1$, and $S = -2$ (Ξ^- hyperon).

Given:

$I_3 = -1/2$

$B = 1$

$S = -2$

Formula:

$Q = I_3 + (B + S) / 2$

Substitute:

$Q = (-1/2) + (1 + (-2)) / 2$

$Q = (-1/2) + (-1/2)$

$Q = -1$

Answer: $Q = -1$ **(Ξ^- hyperon)**

Problem 5

Find Q for a particle with $I_3 = -1/2$, $B = 0$, and $S = +1$ (K^0 meson). [Note: the book prints $I_3 = 0$ here; for K^0 the correct value is $I_3 = -1/2$.]

Given:

$I_3 = -1/2$

$B = 0$

$S = +1$

Formula:

$Q = I_3 + (B + S) / 2$

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Substitute:

$Q = (-1/2) + (0 + 1) / 2$

$Q = (-1/2) + (+1/2)$

$Q = 0$

Answer: $Q = 0$ **(K^0 meson)**

QUESTION 6

6. Radioactive Activity: A radioactive sample has an initial activity of 800 Bq and a half-life of 5 days. Find its activity after 15 days.

Given:

$A_0 = 800$ Bq

$T_{1/2} = 5$ days

$t = 15$ days

Formula:

$n = t / T_{1/2}$

$A = A_0 / 2^n$

Substitute:

$n = 15 / 5 = 3$ half-lives

$A = 800 / 2^3$

$A = 800 / 8$

$A = 100$ Bq

Answer: A = 100 Bq

PRACTICE PROBLEM

Problem 1

A sample's initial activity is 1600 Bq and its half-life is 2 hours. Find the activity after 8 hours.

Given:

$A_0 = 1600$ Bq

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$T_{1/2}$ = 2 hours

t = 8 hours

Formula:

$n = t / T_{1/2}$

$A = A_o / 2^n$

Substitute:

n = 8 / 2 = 4 half-lives

$A = 1600 / 2^4$

A = 1600 / 16

A = 100 Bq

Answer: A = **100 Bq**

Problem 2

A sample's initial activity is 400 mCi and its half-life is 10 days. Find the activity after 40 days.

Given:

A_o = 400 mCi

$T_{1/2}$ = 10 days

t = 40 days

Formula:

$n = t / T_{1/2}$

$A = A_o / 2^n$

Substitute:

n = 40 / 10 = 4 half-lives

$A = 400 / 2^4$

A = 400 / 16

A = 25 mCi

Answer: A = **25 mCi**

Problem 3

A sample's initial activity is 64 Bq and its half-life is 3 minutes. Find the activity after 18 minutes.

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Given:

$A_0 = 64$ Bq

$T_{1/2} = 3$ minutes

$t = 18$ minutes

Formula:

$n = t / T_{1/2}$

$A = A_0 / 2^n$

Substitute:

$n = 18 / 3 = 6$ half-lives

$A = 64 / 2^6$

$A = 64 / 64$

$A = 1$ Bq

Answer: $A =$ **1 Bq**

Problem 4

A sample's initial activity is 1000 Bq and its half-life is 1 year. Find the activity after 3 years.

Given:

$A_0 = 1000$ Bq

$T_{1/2} = 1$ year

$t = 3$ years

Formula:

$n = t / T_{1/2}$

$A = A_0 / 2^n$

Substitute:

$n = 3 / 1 = 3$ half-lives

$A = 1000 / 2^3$

$A = 1000 / 8$

$A = 125$ Bq

Answer: $A =$ **125 Bq**

Problem 5

A sample's initial activity is 512 Ci and its half-life is 6 hours. Find the activity after 24 hours.

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Given:

$A_0 = 512$ Ci

$T_{1/2} = 6$ hours

$t = 24$ hours

Formula:

$n = t / T_{1/2}$

$A = A_0 / 2^n$

Substitute:

$n = 24 / 6 = 4$ half-lives

$A = 512 / 2^4$

$A = 512 / 16$

$A = 32$ Ci

Answer: $A =$ **32 Ci**

QUESTION 7

7. Alpha Decay Energy: In the alpha decay of a Radium nucleus ($^{226}Ra \rightarrow {}^{222}Rn + \alpha$), the total Q-value released is 4.87 MeV. Calculate the kinetic energy of the emitted alpha particle.

Given:

$A = 226$ (parent)

$Q = 4.87$ MeV

Formula:

$K_\alpha = Q \times (A - 4) / A$

Substitute:

$K_\alpha = 4.87 \times (226 - 4) / 226$

$K_\alpha = 4.87 \times 222 / 226$

$K_\alpha = 4.87 \times 0.9823$

$K_\alpha = 4.784$ MeV

Answer: $K_\alpha = 4.78$ MeV

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PRACTICE PROBLEM

Problem 1

A nucleus with A = 210 emits an α-particle with Q = 5.40 MeV. Find K_α.

Given:

A = 210

Q = 5.40 MeV

Formula:

$K_\alpha = Q \times (A - 4) / A$

Substitute:

$K_\alpha = 5.40 \times (210 - 4) / 210$

$K_\alpha = 5.40 \times 206 / 210$

$K_\alpha = 5.40 \times 0.9810$

$K_\alpha = 5.297$ MeV

Answer: $K_\alpha =$ **5.30 MeV**

Problem 2

A nucleus with A = 238 emits an α-particle with Q = 4.27 MeV. Find K_α.

Given:

A = 238

Q = 4.27 MeV

Formula:

$K_\alpha = Q \times (A - 4) / A$

Substitute:

$K_\alpha = 4.27 \times (238 - 4) / 238$

$K_\alpha = 4.27 \times 234 / 238$

$K_\alpha = 4.27 \times 0.9832$

$K_\alpha = 4.198$ MeV

Answer: $K_\alpha =$ **4.20 MeV**

Problem 3

A nucleus with A = 200 emits an α-particle with Q = 6.00 MeV. Find K_α.

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Given:

A = 200

Q = 6.00 MeV

Formula:

$K_\alpha = Q \times (A - 4) / A$

Substitute:

$K_\alpha = 6.00 \times (200 - 4) / 200$

$K_\alpha = 6.00 \times 196 / 200$

$K_\alpha = 6.00 \times 0.9800$

$K_\alpha = 5.880$ MeV

Answer: K_α = **5.88 MeV**

Problem 4

A nucleus with A = 240 emits an α-particle with Q = 5.00 MeV. Find K_α.

Given:

A = 240

Q = 5.00 MeV

Formula:

$K_\alpha = Q \times (A - 4) / A$

Substitute:

$K_\alpha = 5.00 \times (240 - 4) / 240$

$K_\alpha = 5.00 \times 236 / 240$

$K_\alpha = 5.00 \times 0.9833$

$K_\alpha = 4.917$ MeV

Answer: K_α = **4.92 MeV**

Problem 5

A nucleus with A = 214 emits an α-particle with Q = 7.80 MeV. Find K_α.

Given:

A = 214

Q = 7.80 MeV

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Formula:

$K_\alpha = Q \times (A - 4) / A$

Substitute:

$K_\alpha = 7.80 \times (214 - 4) / 214$

$K_\alpha = 7.80 \times 210 / 214$

$K_\alpha = 7.80 \times 0.9813$

$K_\alpha = 7.654$ MeV

Answer: K_α = **7.65 MeV**

QUESTION 8

8. Mean Life: The half-life of a radioactive nucleus is 69.3 days. Calculate the mean life (τ) of the nucleus.

Given:

$T_{1/2} = 69.3$ days

Formula:

$\tau = 1 / \lambda = T_{1/2} / 0.693$

Substitute:

$\tau = 69.3 / 0.693$

$\tau = 100$ days

Answer: τ = 100 days

PRACTICE PROBLEM

Problem 1

The half-life of a nucleus is 13.86 hours. Calculate the mean life.

Given:

$T_{1/2} = 13.86$ hours

Formula:

$\tau = 1 / \lambda = T_{1/2} / 0.693$

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Substitute:

$\tau = 13.86 / 0.693$

$\tau = 20$ hours

Answer: τ = 20 hours

Problem 2

The half-life of a nucleus is 0.693 seconds. Calculate the mean life.

Given:

$T_{1/2} = 0.693$ seconds

Formula:

$\tau = 1 / \lambda = T_{1/2} / 0.693$

Substitute:

$\tau = 0.693 / 0.693$

$\tau = 1$ second

Answer: τ = 1 second

Problem 3

The half-life of a nucleus is 27.72 years. Calculate the mean life.

Given:

$T_{1/2} = 27.72$ years

Formula:

$\tau = 1 / \lambda = T_{1/2} / 0.693$

Substitute:

$\tau = 27.72 / 0.693$

$\tau = 40$ years

Answer: τ = 40 years

Problem 4

The half-life of a nucleus is 34.65 minutes. Calculate the mean life.

Given:

$T_{1/2} = 34.65$ minutes

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Formula:

$\tau = 1 / \lambda = T_{1/2} / 0.693$

Substitute:

$\tau = 34.65 / 0.693$

$\tau = 50$ minutes

Answer: τ = 50 minutes

Problem 5

The half-life of a nucleus is 6930 years. Calculate the mean life.

Given:

$T_{1/2} = 6930$ years

Formula:

$\tau = 1 / \lambda = T_{1/2} / 0.693$

Substitute:

$\tau = 6930 / 0.693$

$\tau = 10000$ years

Answer: τ = 10000 years

QUESTION 9

9. Nuclear Shell Model: According to the nuclear shell model, what is the maximum number of nucleons (of one kind) that can be accommodated up to the fully closed $1d_{5/2}$ subshell?

Given:

Subshells filled up to $1d_{5/2}$

Formula:

Capacity of each subshell = 2j + 1

Filling order: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$ …

Substitute:

$1s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1p_{3/2} \Rightarrow 2(3/2) + 1 = 4$

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$1p_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1d_{5/2} \Rightarrow 2(5/2) + 1 = 6$

Total = 2 + 4 + 2 + 6

Total = 14 nucleons

Answer: Total = 14 nucleons

PRACTICE PROBLEM

Problem 1

Calculate the maximum number of nucleons up to the $1p_{3/2}$ subshell.

Given:

Subshells filled up to $1p_{3/2}$

Formula:

Capacity of each subshell = 2j + 1

Substitute:

$1s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1p_{3/2} \Rightarrow 2(3/2) + 1 = 4$

Total = 2 + 4

Total = 6 nucleons

Answer: Total = **6 nucleons**

Problem 2

Calculate the maximum number of nucleons up to the $1p_{1/2}$ subshell.

Given:

Subshells filled up to $1p_{1/2}$

Formula:

Capacity of each subshell = 2j + 1

Substitute:

$1s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1p_{3/2} \Rightarrow 2(3/2) + 1 = 4$

$1p_{1/2} \Rightarrow 2(1/2) + 1 = 2$

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Total = 2 + 4 + 2

Total = 8 nucleons

Answer: Total = **8 nucleons (magic number 8)**

Problem 3

Calculate the maximum number of nucleons up to the $2s_{1/2}$ subshell.

Given:

Subshells filled up to $2s_{1/2}$

Formula:

Capacity of each subshell = 2j + 1

Substitute:

$1s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1p_{3/2} \Rightarrow 2(3/2) + 1 = 4$

$1p_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1d_{5/2} \Rightarrow 2(5/2) + 1 = 6$

$2s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

Total = 2 + 4 + 2 + 6 + 2

Total = 16 nucleons

Answer: Total = **16 nucleons**

Problem 4

Calculate the maximum number of nucleons up to the $1d_{3/2}$ subshell.

Given:

Subshells filled up to $1d_{3/2}$

Formula:

Capacity of each subshell = 2j + 1

Substitute:

$1s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1p_{3/2} \Rightarrow 2(3/2) + 1 = 4$

$1p_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1d_{5/2} \Rightarrow 2(5/2) + 1 = 6$

$2s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

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$1d_{3/2} \Rightarrow 2(3/2) + 1 = 4$

Total = 2 + 4 + 2 + 6 + 2 + 4

Total = 20 nucleons

Answer: Total = **20 nucleons (magic number 20)**

Problem 5

Calculate the maximum number of nucleons up to the $1f_{7/2}$ subshell.

Given:

Subshells filled up to $1f_{7/2}$

Formula:

Capacity of each subshell = 2j + 1

Substitute:

$1s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1p_{3/2} \Rightarrow 2(3/2) + 1 = 4$

$1p_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1d_{5/2} \Rightarrow 2(5/2) + 1 = 6$

$2s_{1/2} \Rightarrow 2(1/2) + 1 = 2$

$1d_{3/2} \Rightarrow 2(3/2) + 1 = 4$

$1f_{7/2} \Rightarrow 2(7/2) + 1 = 8$

Total = 2 + 4 + 2 + 6 + 2 + 4 + 8

Total = 28 nucleons

Answer: Total = **28 nucleons (magic number 28)**

QUESTION 10

10. Hypercharge: A particle belongs to the baryon family (B = 1). It possesses strangeness S = −1 and charm C = 0. Calculate its hypercharge (Y).

Given:

B = 1

S = −1

C = 0

B′ = T = 0

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Formula:

$Y = B + S + C + B' + T$

(B' = bottomness, T = topness; both zero here)

Substitute:

$Y = 1 + (-1) + 0$

$Y = 0$

Answer: Y = 0

PRACTICE PROBLEM

Problem 1

Find the hypercharge of a proton (B = 1, S = 0, C = 0).

Given:

$B = 1$

$S = 0$

$C = 0$

$B' = T = 0$

Formula:

$Y = B + S + C + B' + T$

Substitute:

$Y = 1 + 0 + 0$

$Y = +1$

Answer: Y = **+1**

Problem 2

Find the hypercharge of an Ω^- hyperon (B = 1, S = −3, C = 0).

Given:

$B = 1$

$S = -3$

$C = 0$

$B' = T = 0$

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Formula:

$Y = B + S + C + B' + T$

Substitute:

$Y = 1 + (-3) + 0$

$Y = -2$

Answer: $Y = \mathbf{-2}$

Problem 3

Find the hypercharge of a K^+ meson ($B = 0$, $S = +1$, $C = 0$).

Given:

$B = 0$

$S = +1$

$C = 0$

$B' = T = 0$

Formula:

$Y = B + S + C + B' + T$

Substitute:

$Y = 0 + 1 + 0$

$Y = +1$

Answer: $Y = \mathbf{+1}$

Problem 4

Find the hypercharge of a D^+ meson ($B = 0$, $S = 0$, $C = +1$).

Given:

$B = 0$

$S = 0$

$C = +1$

$B' = T = 0$

Formula:

$Y = B + S + C + B' + T$

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Substitute:

$Y = 0 + 0 + 1$

$Y = +1$

Answer: $Y = \mathbf{+1}$

Problem 5

Find the hypercharge of a Ξ^0 hyperon ($B = 1$, $S = -2$, $C = 0$).

Given:

$B = 1$

$S = -2$

$C = 0$

$B' = T = 0$

Formula:

$Y = B + S + C + B' + T$

Substitute:

$Y = 1 + (-2) + 0$

$Y = -1$

Answer: $Y = \mathbf{-1}$

QUESTION 11

11. Q-Value of a Nuclear Reaction: In a nuclear reaction X + Y → W + Z, the total mass of the reactants is 10.012 amu and the total mass of the products is 10.008 amu. Calculate the Q-value of the reaction in MeV. (1 amu = 931.5 MeV)

Given:

$M_{reactants} = 10.012$ amu

$M_{products} = 10.008$ amu

Formula:

$\Delta m = M_{reactants} - M_{products}$

$Q = \Delta m \times 931.5$ MeV/amu

Substitute:

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$\Delta m = 10.012 - 10.008 = 0.004$ amu

$Q = 0.004 \times 931.5$

$Q = 3.726$ MeV

Answer: Q = 3.726 MeV (Exothermic)

PRACTICE PROBLEM

Problem 1

Reactant mass = 14.003 amu, product mass = 14.000 amu. Find the Q-value.

Given:

M_reactants = 14.003 amu

M_products = 14.000 amu

Formula:

Δm = M_reactants − M_products

$Q = \Delta m \times 931.5$ MeV/amu

Substitute:

$\Delta m = 14.003 - 14.000 = 0.003$ amu

$Q = 0.003 \times 931.5$

$Q = 2.7945$ MeV

Answer: Q = **2.7945 MeV (Exothermic, Q > 0)**

Problem 2

Reactant mass = 8.020 amu, product mass = 8.025 amu. Find the Q-value.

Given:

M_reactants = 8.020 amu

M_products = 8.025 amu

Formula:

Δm = M_reactants − M_products

$Q = \Delta m \times 931.5$ MeV/amu

Substitute:

$\Delta m = 8.020 - 8.025 = -0.005$ amu

$Q = (-0.005) \times 931.5$

$Q = -4.6575$ MeV

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Answer: Q = **−4.6575 MeV (Endothermic, Q < 0)**

Problem 3

Reactant mass = 5.012 amu, product mass = 5.002 amu. Find the Q-value.

Given:

M_reactants = 5.012 amu

M_products = 5.002 amu

Formula:

Δm = M_reactants − M_products

Q = Δm × 931.5 MeV/amu

Substitute:

Δm = 5.012 − 5.002 = 0.010 amu

Q = 0.010 × 931.5

Q = 9.315 MeV

Answer: Q = **9.315 MeV (Exothermic, Q > 0)**

Problem 4

Reactant mass = 16.000 amu, product mass = 15.991 amu. Find the Q-value.

Given:

M_reactants = 16.000 amu

M_products = 15.991 amu

Formula:

Δm = M_reactants − M_products

Q = Δm × 931.5 MeV/amu

Substitute:

Δm = 16.000 − 15.991 = 0.009 amu

Q = 0.009 × 931.5

Q = 8.3835 MeV

Answer: Q = **8.3835 MeV (Exothermic, Q > 0)**

Problem 5

Reactant mass = 12.015 amu, product mass = 12.011 amu. Find the Q-value.

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Given:

M_reactants = 12.015 amu

M_products = 12.011 amu

Formula:

Δm = M_reactants − M_products

Q = Δm × 931.5 MeV/amu

Substitute:

Δm = 12.015 − 12.011 = 0.004 amu

Q = 0.004 × 931.5

Q = 3.726 MeV

Answer: Q = **3.726 MeV (Exothermic, Q > 0)**

QUESTION 12

12. Coulomb Energy (Liquid Drop Model): Using the semi-empirical mass formula, calculate the Coulomb energy term for a Calcium-40 nucleus ($^{40}_{20}Ca$). Assume the Coulomb constant a_C = 0.70 MeV.

Given:

Z = 20

A = 40

a_C = 0.70 MeV

Formula:

E_C = a_C Z(Z − 1) / A^(1/3)

Substitute:

E_C = 0.70 × 20(20 − 1) / (40)^(1/3)

E_C = 0.70 × (20 × 19) / 3.42

E_C = 0.70 × 380 / 3.42

E_C = 0.70 × 111.11

E_C = 77.78 MeV

Answer: E_C ≈ 77.78 MeV

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PRACTICE PROBLEM

Problem 1

Calculate the Coulomb term for $^{16}_{8}O$ (a_C = 0.70 MeV).

Given:

Z = 8

A = 16

a_C = 0.70 MeV

(A)^(1/3) = (16)^(1/3) = 2.520

Formula:

E_C = a_C Z(Z − 1) / A^(1/3)

Substitute:

E_C = 0.70 × 8(8 − 1) / (16)^(1/3)

E_C = 0.70 × (8 × 7) / 2.520

E_C = 0.70 × 56 / 2.520

E_C = 0.70 × 22.22

E_C = 15.56 MeV

Answer: E_C ≈ **15.56 MeV**

Problem 2

Calculate the Coulomb term for $^{27}_{13}Al$ (a_C = 0.70 MeV).

Given:

Z = 13

A = 27

a_C = 0.70 MeV

(A)^(1/3) = (27)^(1/3) = 3

Formula:

E_C = a_C Z(Z − 1) / A^(1/3)

Substitute:

E_C = 0.70 × 13(13 − 1) / (27)^(1/3)

E_C = 0.70 × (13 × 12) / 3

E_C = 0.70 × 156 / 3

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$E_C = 0.70 \times 52.00$

$E_C = 36.40$ MeV

Answer: $E_C \approx$ **36.40 MeV**

Problem 3

Calculate the Coulomb term for $^{56}_{26}Fe$ ($a_C = 0.70$ MeV).

Given:

$Z = 26$

$A = 56$

$a_C = 0.70$ MeV

$(A)^{(1/3)} = (56)^{(1/3)} = 3.826$

Formula:

$E_C = a_C Z(Z - 1) / A^{(1/3)}$

Substitute:

$E_C = 0.70 \times 26(26 - 1) / (56)^{(1/3)}$

$E_C = 0.70 \times (26 \times 25) / 3.826$

$E_C = 0.70 \times 650 / 3.826$

$E_C = 0.70 \times 169.89$

$E_C = 118.92$ MeV

Answer: $E_C \approx$ **118.92 MeV**

Problem 4

Calculate the Coulomb term for $^{64}_{30}Zn$ ($a_C = 0.70$ MeV).

Given:

$Z = 30$

$A = 64$

$a_C = 0.70$ MeV

$(A)^{(1/3)} = (64)^{(1/3)} = 4$

Formula:

$E_C = a_C Z(Z - 1) / A^{(1/3)}$

Substitute:

$E_C = 0.70 \times 30(30 - 1) / (64)^{(1/3)}$

$E_C = 0.70 \times (30 \times 29) / 4$

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E_C = 0.70 × 870 / 4

E_C = 0.70 × 217.50

E_C = 152.25 MeV

Answer: E_C ≈ **152.25 MeV**

Problem 5

Calculate the Coulomb term for $^{208}_{82}Pb$ (a_C = 0.70 MeV).

Given:

Z = 82

A = 208

a_C = 0.70 MeV

(A)^(1/3) = (208)^(1/3) = 5.925

Formula:

E_C = a_C Z(Z − 1) / A^(1/3)

Substitute:

E_C = 0.70 × 82(82 − 1) / (208)^(1/3)

E_C = 0.70 × (82 × 81) / 5.925

E_C = 0.70 × 6642 / 5.925

E_C = 0.70 × 1121.01

E_C = 784.71 MeV

Answer: E_C ≈ **784.71 MeV**

QUESTION 13

13. Range of Nuclear Force (Yukawa Theory): Calculate the approximate range (R) of the strong nuclear force if the rest mass of the exchanged pion (π) is 140 MeV/c². (Given *ħ*c ≈ 197 MeV·fm)

Given:

mc^2 = 140 MeV

*ħ*c = 197 MeV·fm

Formula:

R ≈ *ħ* / (mc)

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$R \approx \hbar c / (mc^2)$

Substitute:

$R \approx 197\ \text{MeV}\cdot\text{fm} / 140\ \text{MeV}$

$R \approx 1.407\ \text{fm}$

Answer: R ≈ 1.41 fm

PRACTICE PROBLEM

Problem 1

Find the range of the force if it is mediated by a hypothetical meson of mass 200 MeV/c².

Given:

$mc^2 = 200\ \text{MeV}$

$\hbar c = 197\ \text{MeV}\cdot\text{fm}$

Formula:

$R \approx \hbar / (mc)$

$R \approx \hbar c / (mc^2)$

Substitute:

$R \approx 197 / 200$

$R \approx 0.985\ \text{fm}$

Answer: R ≈ **0.985 fm**

Problem 2

Find the range of the force if it is mediated by a ρ-meson of mass 770 MeV/c².

Given:

$mc^2 = 770\ \text{MeV}$

$\hbar c = 197\ \text{MeV}\cdot\text{fm}$

Formula:

$R \approx \hbar / (mc)$

$R \approx \hbar c / (mc^2)$

Substitute:

$R \approx 197 / 770$

$R \approx 0.2558\ \text{fm}$

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Answer: R ≈ **0.256 fm**

Problem 3

Find the range of the force if it is mediated by a hypothetical meson of mass 100 MeV/c².

Given:

$mc^2 = 100$ MeV

$\hbar c = 197$ MeV·fm

Formula:

$R \approx \hbar / (mc)$

$R \approx \hbar c / (mc^2)$

Substitute:

R ≈ 197 / 100

R ≈ 1.97 fm

Answer: R ≈ **1.970 fm**

Problem 4

Find the range of the force if it is mediated by a hypothetical meson of mass 400 MeV/c².

Given:

$mc^2 = 400$ MeV

$\hbar c = 197$ MeV·fm

Formula:

$R \approx \hbar / (mc)$

$R \approx \hbar c / (mc^2)$

Substitute:

R ≈ 197 / 400

R ≈ 0.4925 fm

Answer: R ≈ **0.493 fm**

Problem 5

Find the range of the force if it is mediated by a kaon (K) of mass 493 MeV/c².

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Given:

$mc^2 = 493$ MeV

$\hbar c = 197$ MeV·fm

Formula:

$R \approx \hbar / (mc)$

$R \approx \hbar c / (mc^2)$

Substitute:

$R \approx 197 / 493$

$R \approx 0.3996$ fm

Answer: $R \approx$ **0.400 fm**

QUESTION 14

14. Secular Equilibrium: In a state of secular equilibrium, the ratio of daughter atoms to parent atoms (N_2/N_1) is 10^{-4}. If the half-life of the parent isotope is 10^8 years, calculate the half-life of the daughter isotope.

Given:

$N_2 / N_1 = 10^{-4}$

$T_1 = 10^8$ years

Formula:

$\lambda_1 N_1 = \lambda_2 N_2$

$N_1 / T_1 = N_2 / T_2$ (since $\lambda = 0.693 / T_{1/2}$)

$T_2 = T_1 \times (N_2 / N_1)$

Substitute:

$T_2 = 10^8 \times 10^{-4}$

$T_2 = 10^4$ years

Answer: $T_2 = 10^4$ years

PRACTICE PROBLEM

Problem 1

Ratio $N_2/N_1 = 10^{-6}$ and the parent half-life is 10^9 years. Find the daughter's half-life.

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Given:

$N_2 / N_1 = 10^{-6}$

$T_1 = 10^9$ years

Formula:

$T_2 = T_1 \times (N_2 / N_1)$

Substitute:

$T_2 = 10^9 \times 10^{-6}$

$T_2 = 10^3$ years

Answer: $T_2 =$ **10^3 years**

Problem 2

Ratio $N_2/N_1 = 10^{-3}$ and the parent half-life is 10^6 years. Find the daughter's half-life.

Given:

$N_2 / N_1 = 10^{-3}$

$T_1 = 10^6$ years

Formula:

$T_2 = T_1 \times (N_2 / N_1)$

Substitute:

$T_2 = 10^6 \times 10^{-3}$

$T_2 = 10^3$ years

Answer: $T_2 =$ **10^3 years**

Problem 3

Ratio $N_2/N_1 = 5 \times 10^{-5}$ and the parent half-life is 2×10^7 years. Find the daughter's half-life.

Given:

$N_2 / N_1 = 5 \times 10^{-5}$

$T_1 = 2 \times 10^7$ years

Formula:

$T_2 = T_1 \times (N_2 / N_1)$

Substitute:

$T_2 = (2 \times 10^7) \times (5 \times 10^{-5})$

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$T_2 = 10 \times 10^2$

$T_2 = 10^3$ years

Answer: $T_2 = \mathbf{10^3}$ **years**

Problem 4

Ratio $N_2/N_1 = 2 \times 10^{-4}$ and the parent half-life is 4×10^8 years. Find the daughter's half-life.

Given:

$N_2 / N_1 = 2 \times 10^{-4}$

$T_1 = 4 \times 10^8$ years

Formula:

$T_2 = T_1 \times (N_2 / N_1)$

Substitute:

$T_2 = (4 \times 10^8) \times (2 \times 10^{-4})$

$T_2 = 8 \times 10^4$

$T_2 = 8 \times 10^4$ years

Answer: $T_2 = \mathbf{8 \times 10^4}$ **years**

Problem 5

Ratio $N_2/N_1 = 10^{-5}$ and the parent half-life is 5×10^{10} years. Find the daughter's half-life.

Given:

$N_2 / N_1 = 10^{-5}$

$T_1 = 5 \times 10^{10}$ years

Formula:

$T_2 = T_1 \times (N_2 / N_1)$

Substitute:

$T_2 = (5 \times 10^{10}) \times (10^{-5})$

$T_2 = 5 \times 10^5$

$T_2 = 5 \times 10^5$ years

Answer: $T_2 = \mathbf{5 \times 10^5}$ **years**

QUESTION 15

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15. Recoil Energy in Alpha Decay: A parent nucleus with mass number A = 200 undergoes alpha decay with a total energy release Q = 5.0 MeV. Calculate the recoil kinetic energy of the heavy daughter nucleus.

Given:

A = 200

Q = 5.0 MeV

Formula:

$K_D = Q \times (4 / A)$

Substitute:

$K_D = 5.0 \times (4 / 200)$

$K_D = 5.0 \times 0.02$

$K_D = 0.1$ MeV

Answer: $K_D = 0.1$ MeV

PRACTICE PROBLEM

Problem 1

A parent with A = 210 decays with Q = 6.3 MeV. Find the recoil energy of the daughter.

Given:

A = 210

Q = 6.3 MeV

Formula:

$K_D = Q \times (4 / A)$

Substitute:

$K_D = 6.3 \times (4 / 210)$

$K_D = 6.3 \times 0.01905$

$K_D = 0.12$ MeV

Answer: $K_D =$ **0.12 MeV**

Problem 2

A parent with A = 240 decays with Q = 4.8 MeV. Find the recoil energy of the daughter.

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Given:

A = 240

Q = 4.8 MeV

Formula:

$K_D = Q \times (4 / A)$

Substitute:

$K_D = 4.8 \times (4 / 240)$

$K_D = 4.8 \times 0.01667$

$K_D = 0.08$ MeV

Answer: $K_D =$ **0.08 MeV**

Problem 3

A parent with A = 226 decays with Q = 5.5 MeV. Find the recoil energy of the daughter.

Given:

A = 226

Q = 5.5 MeV

Formula:

$K_D = Q \times (4 / A)$

Substitute:

$K_D = 5.5 \times (4 / 226)$

$K_D = 5.5 \times 0.01770$

$K_D = 0.0973$ MeV

Answer: $K_D =$ **0.0973 MeV**

Problem 4

A parent with A = 180 decays with Q = 4.5 MeV. Find the recoil energy of the daughter.

Given:

A = 180

Q = 4.5 MeV

Formula:

$K_D = Q \times (4 / A)$

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Substitute:

$K_D = 4.5 \times (4 / 180)$

$K_D = 4.5 \times 0.02222$

$K_D = 0.1$ MeV

Answer: $K_D =$ **0.1 MeV**

Problem 5

A parent with A = 235 decays with Q = 5.0 MeV. Find the recoil energy of the daughter.

Given:

A = 235

Q = 5.0 MeV

Formula:

$K_D = Q \times (4 / A)$

Substitute:

$K_D = 5.0 \times (4 / 235)$

$K_D = 5.0 \times 0.01702$

$K_D = 0.0851$ MeV

Answer: $K_D =$ **0.0851 MeV**

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