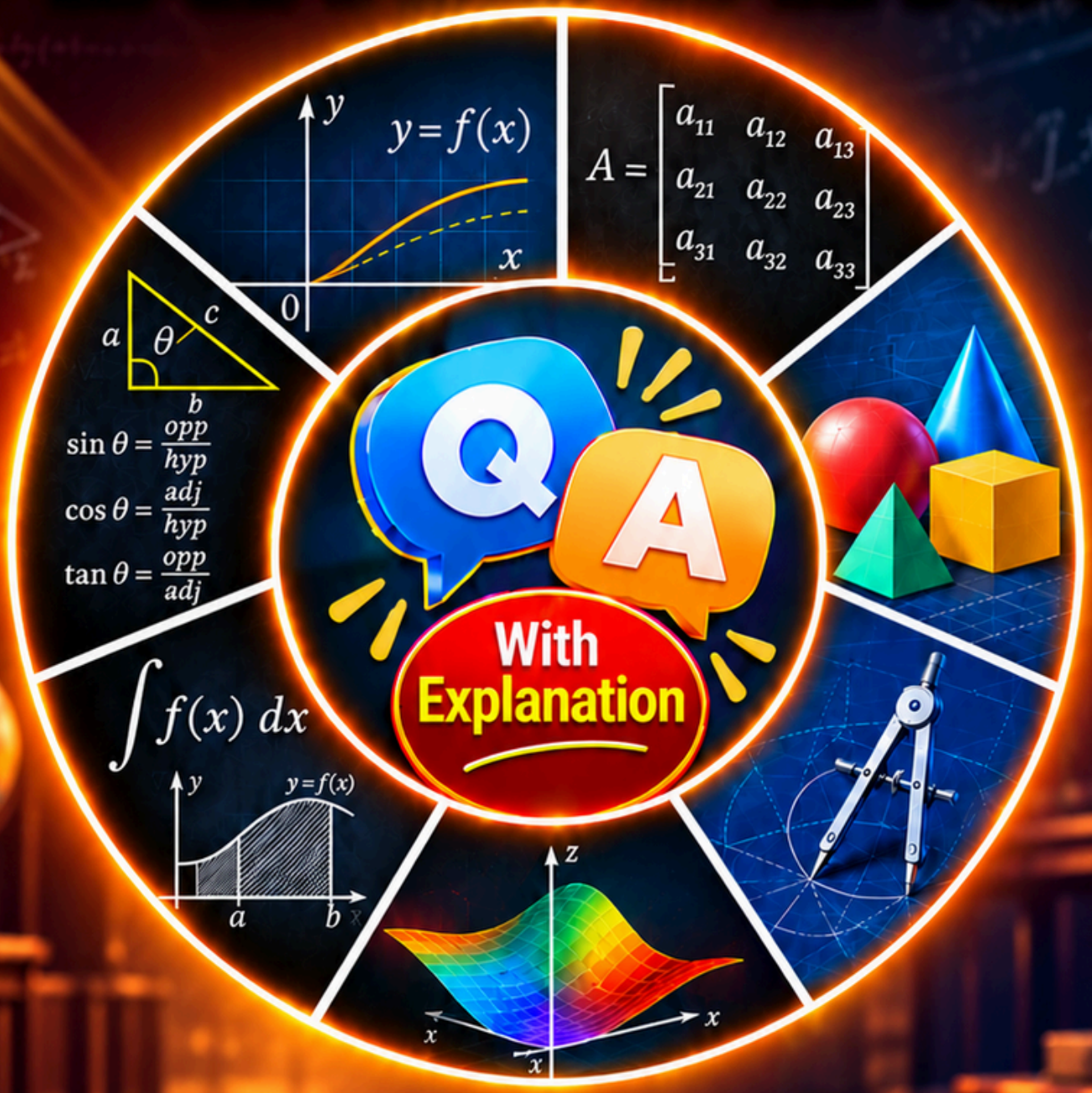
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Mathematics

Test

PART - A

1. The set $\mathbb{R} \setminus \mathbb{Q}$ of all irrational numbers is :

(a) finite

(b) uncountable

(c) bounded

(d) empty

Answer: B. Uncountable

Explanation:
The set $\mathbb{R} \setminus \mathbb{Q}$ represents all real numbers that are not rational numbers, which are called irrational numbers. Examples include $\sqrt{2}$, $\sqrt{3}$, π, and e. The set of irrational numbers cannot be listed one by one using natural numbers. Therefore, the set of irrational numbers is uncountable.

2. If $a_n = \sqrt{n^2 + 5n} - n$ then

$$\lim_{n\to\infty} a_n = ____.$$

(a) $\frac{5}{2}$

(b) 1

(c) 0

(d) ∞

Answer: A. $\frac{5}{2}$

Explanation:
Given $a_n = \sqrt{n^2 + 5n} - n$. To find the limit, rationalize the expression:

$$a_n = \frac{[\sqrt{n^2 + 5n} - n][\sqrt{n^2 + 5n} + n]}{\sqrt{n^2 + 5n} + n}$$

$$= \frac{5n}{\sqrt{n^2 + 5n} + n}$$

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Dividing numerator and denominator by n:

$$= \frac{5}{\sqrt{1+5/n}+1}$$

As $n \to \infty$, $5/n \to 0$.

Therefore,

$$\lim a_n = \frac{5}{1+1} = \frac{5}{2}.$$

3. If $f : [a, b] \to \mathbb{R}$ is a continuous function then the function f is :

(a) constant function

(b) bounded function

(c) increasing function

(d) decreasing function

Answer: B. Bounded function

Explanation:
A continuous function defined on a closed and bounded interval $[a, b]$ must be bounded. This follows from the Extreme Value Theorem. It means that the function has both a maximum value and a minimum value somewhere in the interval. Hence, there exists a finite number M such that $|f(x)| \leq M$ for every x in $[a, b]$.

4. The function

$$f(x) = \begin{cases} \dfrac{x^3-8}{x-2}, & x \neq 2, \\ 3, & x = 2 \end{cases}$$

is :

(a) discontinuous at $x = 2$

(b) continuous only at $x = 2$

(c) continuous

(d) discontinuous at $x = 0$

Answer: A. Discontinuous at $x = 2$

Explanation:
For $x \neq 2$,

$$f(x) = \frac{x^3-8}{x-2}$$

Using $x^3 - 8 = (x-2)(x^2+2x+4)$,

$$f(x) = x^2 + 2x + 4.$$

Therefore,

$$\lim_{x\to 2} f(x) = 2^2 + 2(2) + 4 = 12.$$

But the given value is $f(2) = 3$.

Since $\lim_{x\to 2} f(x) \neq f(2)$, the function is discontinuous at $x = 2$.

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5. Which one of the following statement is wrong ?

(a) uniformly continuous function is continuous

(b) continuous function defined on a closed, bounded interval is uniformly continuous

(c) continuous function is uniformly continuous

(d) continuous function defined on a closed bounded interval is a bounded function

Answer: C. Continuous function is uniformly continuous

Explanation:
This statement is wrong because every continuous function need not be uniformly continuous. Uniform continuity is a stronger condition than ordinary continuity. For example, $f(x) = 1/x$ is continuous on the interval $(0, 1)$, but it is not uniformly continuous there. However, every continuous function on a closed and bounded interval $[a, b]$ is uniformly continuous.

6. The series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

is:

(a) Absolutely convergent

(b) Divergent

(c) Oscillatory

(d) Conditionally convergent

Answer: D. Conditionally convergent

Explanation:
The given series is

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots .$$

This is the alternating harmonic series. By the Alternating Series Test, it converges because

$$\frac{1}{n+1} < \frac{1}{n}$$

and

$$\lim_{n\to\infty} \frac{1}{n} = 0.$$

However, the corresponding series of absolute values is

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n},$$

which is the harmonic series and diverges.

Therefore, the given series converges but does not converge absolutely. Hence, it is conditionally convergent.

7. The sequence of functions $\{f_n\}$ where $f_n(x) = x^n,\ 0 \le x \le 1$:

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(a) converges pointwise to $f(x) = 0$ $(0 \le x < 1)$, $f(1) = 1$

(b) converges uniformly to $f(x) = 0$ $(0 \le x < 1)$, $f(1) = 1$

(c) converges pointwise to $f(x) = 0$, $(0 \le x \le 1)$

(d) converges uniformly to $f(x) = 0$, $(0 \le x \le 1)$

Answer: A. Converges pointwise to $f(x) = 0$ for $0 \le x < 1$ and $f(1) = 1$

Explanation:
Given $f_n(x) = x^n$ on $[0, 1]$. For every fixed x with $0 \le x < 1$, x^n approaches 0 as n approaches infinity. But when $x = 1$, we have $1^n = 1$ for every n. Therefore, the pointwise limit is $f(x) = 0$ for $0 \le x < 1$ and $f(1) = 1$. The convergence is not uniform because values of x very close to 1 can make x^n remain close to 1 even for large n.

8. For the Gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

which one of the following is false ?

(a) Γ is convergent for $x > 0$

(b) $\Gamma(x)\Gamma(1-x) = \dfrac{1}{\sin \pi}$, $0 < x < 1$

(c) $\Gamma(n+1) = n\Gamma(n)$

(d) $\Gamma\left(\dfrac{1}{2}\right) = \sqrt{\pi}$

Answer: B. $\Gamma(x)\Gamma(1-x) = 1/\sin \pi$

Explanation:
The correct reflection formula for the Gamma function is

$$\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin(\pi x)}, \quad \text{for } 0 < x < 1.$$

The given statement is missing π in the numerator and x inside the sine term. Therefore, option B is false. Also, $\Gamma(1/2) = \sqrt{\pi}$ and the Gamma function converges for $x > 0$.

9. If $y = x^x$ then $\dfrac{dy}{dx}$ is :

(a) $x^{x-1} \ln x$

(b) $x^x(1 + \ln x)$

(c) x^x

(d) $\dfrac{x^{x-1}}{\ln x}$

Answer: B. $x^x(1 + \ln x)$

Explanation:
Given $y = x^x$. Since both the base and exponent contain x, logarithmic differentiation is used.

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Taking logarithm,

$$\ln y = x \ln x$$

Differentiating both sides,

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

Therefore,

$$\frac{dy}{dx} = y(1 + \ln x)$$

Since $y = x^x$,

$$\frac{dy}{dx} = x^x(1 + \ln x).$$

Hence, option B is correct.

10. For the characteristic function χ of the rational numbers in $[0, 1]$, and for any subdivision σ of $[0, 1]$, the value of the upper Riemann sum $u[\chi, \sigma]$ is :

(a) $\frac{1}{n}$

(b) 0

(c) $\frac{1}{2n}$

(d) 1

Answer: D. 1

Explanation:
The characteristic function of rational numbers has value 1 at rational numbers and 0 at irrational numbers. Every interval, however small, contains both rational and irrational numbers. Therefore, the supremum of the function in every subinterval is 1. Hence, for any subdivision of $[0, 1]$, the upper Riemann sum is the total interval length multiplied by 1, which gives 1.

11. $e^z = -1$ if $z =$ ____ .

(a) $-3\pi i$

(b) $-4\pi i$

(c) $2\pi i$

(d) $-2\pi i$

Answer: A. $-3\pi i$

Explanation:
For a complex number z, $e^z = -1$ when z is an odd multiple of πi. In general, $z = (2n + 1)\pi i$, where n is any integer. Since $-3\pi i$ is an odd multiple of πi, $e^{-3\pi i} = \cos(-3\pi) + i \sin(-3\pi) = -1$. Hence, option A is correct.

12. $w = f(z)$ is analytic in a domain D containing z_0 and $w = f(z)$ is conformal at z_0 if :

(a) $f(z_0) = 0$

(b) $f(z_0) \neq 0$

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(c) $f'(z_0) = 0$

(d) $f'(z_0) \neq 0$

Answer: D. $f'(z_0) \neq 0$

Explanation:
An analytic function $f(z)$ is conformal at a point z_0 when its derivative exists and is non-zero at that point. Since f is already given to be analytic in D, the required additional condition is $f'(z_0) \neq 0$. This ensures that angles between curves are preserved locally at z_0.

13.

$$\int_C \frac{dz}{z} = \text{----}$$

where $C : x = \cos t,\ y = \sin t,\ 0 \leq t \leq 2\pi$

(a) $\dfrac{i}{2\pi}$

(b) $\dfrac{1}{2\pi i}$

(c) $2\pi i$

(d) $\dfrac{1}{2\pi}$

Answer: C. $2\pi i$

Explanation:
The curve C given by $x = \cos t$ and $y = \sin t$, with $0 \leq t \leq 2\pi$, is the unit circle traversed once counterclockwise around the origin. The function $1/z$ has a simple pole at $z = 0$ inside the curve. By the standard contour integral result,

$$\int_C \frac{dz}{z} = 2\pi i.$$

Hence, option C is correct.

14. The maximum value of

$$\left|z^2 + 5z - 1\right|$$

on

$$|z| \leq 1$$

is:

(a) 7

(b) 5

(c) $\sqrt{29}$

(d) 3

Answer: C. $\sqrt{29}$

Explanation:

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Let

$$f(z) = z^2 + 5z - 1.$$

Since f is a non-constant polynomial, by the Maximum Modulus Principle, the maximum of $|f(z)|$ on the closed unit disk $|z| \leq 1$ occurs on the boundary

$$|z| = 1.$$

Put

$$z = e^{i\theta}.$$

Then

$$f(z) = e^{2i\theta} + 5e^{i\theta} - 1.$$

Hence,

$$|f(z)|^2 = \left(e^{2i\theta} + 5e^{i\theta} - 1\right)\left(e^{-2i\theta} + 5e^{-i\theta} - 1\right).$$

On simplification,

$$|f(z)|^2 = 27 - 2\cos 2\theta.$$

Since

$$-1 \leq \cos 2\theta \leq 1,$$

the maximum occurs when

$$\cos 2\theta = -1.$$

Therefore,

$$|f(z)|^2_{\max} = 27 + 2 = 29.$$

Thus,

$$\boxed{|f(z)|_{\max} = \sqrt{29}}.$$

For example, when

$$z = i,$$

we obtain

$$f(i) = i^2 + 5i - 1 = -2 + 5i,$$

so

$$|f(i)| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}.$$

15. If $f(z)$ is continuous and

$$\int_C f(z)\,dz$$

is independent of the path C in a domain D then $f(z)$ has :

(a) antiderivative everywhere in D

(b) no antiderivative in D

(c) discontinuity in D

(d) maximum absolute value in D

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Answer: A. Antiderivative everywhere in D

Explanation:
If $f(z)$ is continuous in a domain D and the contour integral $\int_C f(z)\,dz$ is independent of the path, then we can define a function $F(z)$ using the integral from a fixed point to z. Path independence ensures that F is well-defined, and $F'(z) = f(z)$. Therefore, f has an antiderivative everywhere in D.

16. The circle of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{z^{k+1}}{k}$$

is:

(a) $|z| = 2$

(b) $|z| = \frac{1}{2}$

(c) $|z| = 1$

(d) $|z| = \frac{1}{4}$

Answer: C. $|z| = 1$

Explanation:
The given power series is

$$\sum \frac{z^{k+1}}{k}.$$

The coefficient essentially behaves like $1/k$. Using the ratio or root test, the radius of convergence is $R = 1$. Therefore, the series converges for $|z| < 1$ and diverges for $|z| > 1$. Hence, its circle of convergence is $|z| = 1$.

17. The function

$$f(z) = \frac{1}{z^2 + 1}$$

is :

(a) Entire function

(b) Meromorphic function

(c) Nowhere analytic function

(d) Bounded entire function

Answer: B. Meromorphic function

Explanation:
The function is

$$f(z) = \frac{1}{z^2 + 1} = \frac{1}{(z - i)(z + i)}.$$

It is analytic everywhere except at $z = i$ and $z = -i$, where it has isolated simple poles. A function that is analytic except at isolated poles is called a meromorphic function. Therefore, option B is correct.

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18. Residue of

$$f(z) = \frac{1}{(z-1)^2(z-3)}$$

at $z = 3$ is :

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{5}$

(d) $\frac{1}{3}$

Answer: B. $\frac{1}{4}$

Explanation:
At $z = 3$, the function

$$f(z) = \frac{1}{(z-1)^2(z-3)}$$

has a simple pole. For a simple pole, the residue is calculated by

$$\text{Residue} = \lim_{z\to 3}[(z-3)f(z)]$$

$$= \lim_{z\to 3} \frac{1}{(z-1)^2}$$

$$= \frac{1}{(3-1)^2}$$

$$= \frac{1}{4}.$$

Hence, option B is correct.

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19. A closed curve $C : z(t)$, $a \le t \le b$ which is simple in the interval (a, b) and $z(a) = z(b)$ is called:

(a) Jordan curve

(b) Closed curve

(c) Smooth curve

(d) Regular curve

Answer: A. Jordan curve

Explanation:
A Jordan curve is a simple closed curve. Here, the curve is simple in the interval (a, b), meaning it does not intersect itself there, and $z(a) = z(b)$, meaning its starting and ending points are the same. Therefore, it satisfies the definition of a Jordan curve.

20. If the sequence $\{f_n\}$ of analytic functions converges uniformly on every compact sets in the domain of definition (say) D, then the limit function (say) $f(z)$ is :

(a) Discontinuous in D

(b) Analytic in D

(c) Meromorphic in D

(d) Constant in D

Answer: B. Analytic in D

Explanation:
If a sequence of analytic functions $\{f_n\}$ converges uniformly on every compact subset of a domain D, then the limit function f is also analytic in D. This important result is known as the Weierstrass theorem on locally uniform convergence of analytic functions. Therefore, option B is correct.

21. The number of 7-sylow subgroups in a finite group G can be :

(a) 7

(b) 5

(c) 8

(d) 4

Answer: C. 8

Explanation:
By Sylow's theorem, the number n_7 of Sylow 7-subgroups must satisfy two conditions: $n_7 \equiv 1 \pmod 7$, and n_7 must divide the remaining part of the order of the group. Among the given options, only 8 satisfies $n_7 \equiv 1 \pmod 7$. Therefore, 8 can be the number of Sylow 7-subgroups.

22. The degree of the splitting field of $f(x) = x^3 - 2$ over the rationals is :

(a) 3

(b) 6

(c) 4

(d) 2

Answer: B. 6

Explanation:
The polynomial $x^3 - 2$ has the roots $\sqrt[3]{2}$, $\omega\sqrt[3]{2}$ and $\omega^2\sqrt[3]{2}$, where ω is a complex cube root of unity. First adjoining $\sqrt[3]{2}$ gives an extension of degree 3 over $\mathbb{Q}$. We must also adjoin ω, which contributes degree 2. Therefore, the degree of the splitting field is $3 \times 2 = 6$.

23. How many roots of the polynomial $x^4 + 4 \in \mathbb{Z}_7[x]$ in $\mathbb{Z}_7$?

(a) 2

(b) 0

(c) 4

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(d) 3

Answer: B. 0

Explanation:
We need the roots of $x^4 + 4 = 0$ in $\mathbb{Z}_7$. This means $x^4 \equiv -4 \equiv 3 \pmod 7$. Checking the elements 0, 1, 2, 3, 4, 5 and 6 modulo 7 shows that none has fourth power congruent to 3. Therefore, the polynomial has no roots in $\mathbb{Z}_7$, and the number of roots is 0.

24. Which of the following statements is true ?

(a) Every orthogonal matrix has determinant 1 or -1

(b) The product of two orthogonal matrices is not orthogonal

(c) Every orthogonal matrix is not invertible

(d) If A is orthogonal matrix then A^{-1} is not an orthogonal matrix

Answer: A. Every orthogonal matrix has determinant 1 or -1

Explanation:
For an orthogonal matrix A, we have $A^T A = I$. Taking determinants gives $\det(A^T)\det(A) = \det(I) = 1$. Since $\det(A^T) = \det(A)$, we obtain $[\det(A)]^2 = 1$. Therefore, $\det(A) = 1$ or -1.

25. The inverse of the matrix

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

is :

(a) $\begin{bmatrix} -2 & -1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(d) $\begin{bmatrix} -2 & -1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Answer: C. $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Explanation:
For a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse is

$$\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

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For the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, the determinant is

$$(1)(4) - (2)(3) = -2.$$

Therefore, the inverse is

$$-\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}.$$

Hence, option C is correct.

26. Which of the following transformation is not linear?

(a) $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(x, y) = 2x + 3y$

(b) $g : \mathbb{R}^3 \to \mathbb{R}$ by $g(x, y, z) = x + y - 2z$

(c) $h : \mathbb{R}^2 \to \mathbb{R}$ by $h(x, y) = 2x + 3y + 4$

(d) $\phi : \mathbb{R}^2 \to \mathbb{R}^3$ by $\phi(x, y) = (x + 2y, 3y, 3x - y)$

Answer: C. $h(x, y) = 2x + 3y + 4$

Explanation:
A linear transformation must map the zero vector to the zero vector. Here,

$$h(0, 0) = 2(0) + 3(0) + 4 = 4.$$

Since $h(0, 0) \neq 0$, the transformation fails an essential condition for linearity. The constant term 4 makes this an affine transformation rather than a linear transformation.

27. The number of abelian groups (upto isomorphism) of order 240 is :

(a) 12

(b) 16

(c) 5

(d) 10

Answer: C. 5

Explanation:
First factorize the order:

$$240 = 2^4 \times 3 \times 5.$$

The number of non-isomorphic abelian groups corresponding to p^n equals the number of partitions of n. The number of partitions of 4 is 5, while the number of partitions of 1 is 1. Therefore, the total number is

$$5 \times 1 \times 1 = 5.$$

Hence, there are 5 abelian groups of order 240 up to isomorphism.

28. The minimal polynomial of $\sqrt{2} + 5$ over the rationals is :

(a) $x^2 - 9x + 5$

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(b) $x^2 - 10x + 13$

(c) $x^2 - 10x + 23$

(d) $x^2 - 10x + 19$

Answer: C. $x^2 - 10x + 23$

Explanation:
Let $\alpha = \sqrt{2} + 5$. Then

$$\alpha - 5 = \sqrt{2}.$$

Squaring both sides,

$$(\alpha - 5)^2 = 2.$$

Expanding,

$$\alpha^2 - 10\alpha + 25 = 2,$$

so

$$\alpha^2 - 10\alpha + 23 = 0.$$

Therefore, the minimal polynomial of $\sqrt{2} + 5$ over the rational numbers is $x^2 - 10x + 23$.

29. Which of the following matrix is not nilpotent?

(a) $\begin{pmatrix} 1 & 3 & -2 \\ 1 & 3 & -2 \\ 1 & 3 & -2 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -3 & 2 \\ 1 & -3 & 2 \\ 1 & -3 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} -2 & 1 & 1 \\ -3 & 1 & 2 \\ -2 & 1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{pmatrix}$

Answer: A

Explanation:
A matrix is nilpotent if some positive power of the matrix becomes the zero matrix. A nilpotent matrix must have all its eigenvalues equal to zero. For matrix A, there is a non-zero eigenvalue 2. In fact, its powers do not become the zero matrix. Therefore, matrix A is not nilpotent, while the other given matrices are nilpotent.

30. Which of the following is a unitary matrix ?

(a) $\begin{bmatrix} i & 1 \\ -1 & -i \end{bmatrix}$

(b) $\begin{pmatrix} 1 & i & 2 \\ 1+i & 3 & 1-i \end{pmatrix}$

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(c) $\frac{1}{2}\begin{pmatrix} 1 & -i & -1+i \\ i & 1 & 1+i \\ 1+i & -1+i & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1+i & 1 \\ 1+i & 1 \end{pmatrix}$

Answer: C

Explanation:
A complex matrix A is unitary when its conjugate transpose A^* satisfies

$$A^*A = I.$$

Equivalently, the rows or columns of the matrix must form an orthonormal set with respect to the complex inner product. On calculating the conjugate transpose and multiplying it by the given matrix in option C, the result is the identity matrix. Therefore, option C is a unitary matrix.

31. If B is the collection of all intervals in the real line of the form

$$[a, b) = \{x/a \leq x < b\},$$

then the topology generated by B is known as :

(a) standard topology

(b) normal topology

(c) limit topology

(d) lower limit topology

Answer: D. Lower limit topology

Explanation:
The collection of all half-open intervals $[a, b)$, where $a < b$, forms a basis for a topology on the real line. The topology generated by these intervals is called the lower limit topology. It is also commonly known as the Sorgenfrey line topology.

32. The lower limit topology on the real line is generated by the collection B of all intervals in the real line of the form :

(a) $[a, b] = \{x/a \leq x \leq b\}$

(b) $[a, b) = \{x/a \leq x < b\}$

(c) $(a, b] = \{x/a < x \leq b\}$

(d) $(a, b) = \{x/a < x < b\}$

Answer: B. $[a, b) = \{x : a \leq x < b\}$

Explanation:
The lower limit topology on $\mathbb{R}$ has a basis consisting of intervals of the form $[a, b)$. Each interval contains its left endpoint a but does not contain its right endpoint b. Therefore, option B gives the correct basis for the lower limit topology.

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33. Let $\{T_\alpha\}$ be a family of topologies on X. Which one of the following is not true ?

(a) $\bigcap T_\alpha$ is a topology on X

(b) $\bigcap T_\alpha$ is not a topology on X

(c) There is a unique smallest topology on X containing all the collections T_α

(d) There is a unique largest topology contained in all T_α

Answer: B. $\bigcap T_\alpha$ is not a topology on X

Explanation:
The intersection of any family of topologies on the same set X is always a topology on X. This is because the intersection still contains the empty set and X and remains closed under arbitrary unions and finite intersections. Therefore, the statement saying that $\bigcap T_\alpha$ is not a topology is false.

34. The collection

$$C = \{[a, b)/a < b,\ a \text{ and } b \text{ rational}\}$$

is a basis that generates :

(a) the lower limit topology

(b) the K-topology

(c) the standard topology

(d) a topology different from the lower limit topology on $\mathbb{R}$

Answer: D. A topology different from the lower limit topology on $\mathbb{R}$

Explanation:
Here the basis contains intervals $[a, b)$ only when both endpoints a and b are rational. This restriction means that it does not generate the usual lower limit topology. For example, a basic lower-limit neighborhood $[\sqrt{2}, b)$ cannot be reproduced locally using basis elements whose left endpoints must be rational. Therefore, it generates a different topology.

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35. The order topology on the set

$$X = \{1, 2\} \times \mathbb{Z}_+$$

in the dictionary order is :

(a) the discrete topology

(b) not the discrete topology

(c) the standard topology

(d) the lower limit topology

Answer: B. Not the discrete topology

Explanation:
Consider $X = \{1, 2\} \times \mathbb{Z}_+$ with dictionary or lexicographic order. Points such as $(1, n)$ can have

immediate neighboring points, but the ordering has special behavior around points like $(2, n)$. In particular, not every singleton can be obtained as an open interval in the order topology. Therefore, the resulting order topology is not discrete.

36. The topology generated by the open rays contains the :

(a) standard topology

(b) lower limit topology

(c) K-topology

(d) order topology

Answer: D. Order topology

Explanation:
In an ordered set, the order topology is generated using open intervals together with suitable open rays at endpoints. Open intervals can be obtained by intersecting appropriate open rays. Therefore, the topology generated by the relevant open rays contains the order topology.

37. If A° and $\overline{A}$ denotes the interior and closure of a set A respectively, then :

(a) $A \subset A^\circ \subset \overline{A}$

(b) $A \subset \overline{A} \subset A^\circ$

(c) $A^\circ \subset A \subset \overline{A}$

(d) $A^\circ \subset \overline{A} \subset A$

Answer: C. $A^\circ \subset A \subset \overline{A}$

Explanation:
The interior A° consists of all interior points of A, so every point of A° belongs to A. The closure $\overline{A}$ contains A together with all its limit points. Therefore, the general relationship is $A^\circ \subseteq A \subseteq \overline{A}$.

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38. Let A° and $\overline{A}$ denotes the interior and closure of a set A. Then :

(a) A is closed if $A = A^\circ$

(b) A is open if $A = \overline{A}$

(c) A is closed if $A = \overline{A}$

(d) A is open if $A^\circ = \overline{A}$

Answer: C. A is closed if $A = \overline{A}$

Explanation:
The closure $\overline{A}$ of a set A is the smallest closed set containing A. A set is closed exactly when it already contains all of its limit points. Therefore, A is closed if and only if A equals its closure $\overline{A}$.

39. Which one of the following statements is not true ?

(a) The point 0 is not a limit point of $A = (0, 1]$

(b) The point 1 is a limit point of $A = (0, 1]$

(c) The point $\frac{1}{2}$ is a limit point of $A = (0, 1]$

(d) Every point of the set $A = (0, 1]$ is a limit point of A

Answer: A. The point 0 is not a limit point of $A = (0, 1]$

Explanation:
This statement is false. Although 0 does not belong to $A = (0, 1]$, every open interval around 0 contains points of A other than 0. For example, every interval $(-\varepsilon, \varepsilon)$ contains positive numbers belonging to $(0, 1]$. Therefore, 0 is a limit point of A.

40. If $\mathbb{Z}_+$ is the set of positive integers, then :

(a) 0 is a limit point of $\mathbb{Z}_+$

(b) 1 is a limit point of $\mathbb{Z}_+$

(c) 0 and 1 are limit points of $\mathbb{Z}_+$

(d) any real number is not a limit point of $\mathbb{Z}_+$

Answer: D. Any real number is not a limit point of $\mathbb{Z}_+$

Explanation:
The positive integers $\mathbb{Z}_+ = \{1, 2, 3, \ldots\}$ are isolated from one another in $\mathbb{R}$. Around every positive integer, we can choose a sufficiently small open interval containing no other positive integer. For a real number not in $\mathbb{Z}_+$, we can also find a small neighborhood containing no positive integers.

41. Every countable set has measure _____ .

(a) zero

(b) one

(c) infinite

(d) equals the cardinality

Answer: A. Zero

Explanation:
Every countable subset of the real numbers has Lebesgue measure zero. A countable set can be written as $\{x_1, x_2, x_3, \ldots\}$, and each individual point has measure zero. Therefore, the measure of the entire countable set is also zero.

42. The cantor-like set $P^{(\alpha)}$ is measurable with measure :

(a) 1

(b) α

(c) $1 + \alpha$

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(d) $1 - \alpha$

Answer: D. $1 - \alpha$

Explanation:
In the construction of the Cantor-like set $P^{(\alpha)}$, intervals having a total length α are removed from the original interval of length 1. Therefore, the measure of the remaining Cantor-like set is $1 - \alpha$. Hence, option D is correct.

43. Which of the following statements are true ?

(i) Every countable set has measure zero

(ii) There exists uncountable sets of zero measure

(iii) Every measurable set is a Borel set

(a) (i), (ii)

(b) (i), (iii)

(c) (ii), (iii)

(d) (i), (ii), (iii)

Answer: A. (i) and (ii)

Explanation:
Statement (i) is true because every countable set has Lebesgue measure zero. Statement (ii) is also true because there are uncountable sets having measure zero, such as the standard Cantor set. Statement (iii) is false because every Borel set is Lebesgue measurable, but not every Lebesgue measurable set is necessarily a Borel set.

44. Let f be a Riemann integrable function on $[a, b]$, and define

$$F(x) = \int_a^x f(t)\, dt.$$

Which one of the following statements is always true?

(a) F is differentiable at every point of $[a, b]$

(b) $F'(x) = f(x)$ at every point of $[a, b]$

(c) F need not be continuous on $[a, b]$

(d) F is continuous, and hence Riemann integrable on $[a, b]$

Answer: D. F is continuous, and hence Riemann integrable on $[a, b]$

Explanation:
Since f is Riemann integrable on $[a, b]$, it is bounded. Thus, there exists $M > 0$ such that

$$|f(t)| \leq M$$

for all $t \in [a, b]$.

For $x, y \in [a, b]$,

$$|F(x) - F(y)| = \left| \int_y^x f(t)\, dt \right| \leq M|x - y|.$$

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Hence F is continuous on $[a, b]$.

Every continuous function on a closed and bounded interval is Riemann integrable. Therefore, F is Riemann integrable on $[a, b]$.

However, F need not be differentiable at every point unless additional conditions, such as continuity of f, are imposed.

Hence, option D is correct.

45. Which of the following statements are true?

(i) If $f_n \to f$ a-u, then $f_n \to f$ in measure

(ii) If $f_n \to f$ a-u, then $f_n \to f$ a-e

(iii) If $f_n \to f$ a-u, then $f_n \to f$ uniformly

(a) (i), (ii)

(b) (ii), (iii)

(c) (i), (iii)

(d) (i), (ii), (iii)

Answer: A. (i) and (ii)

Explanation:

Here, "a-u" means **almost uniform convergence** and "a-e" means **almost everywhere convergence**.

If $f_n \to f$ almost uniformly, then for every $\varepsilon > 0$, there exists a measurable set E with arbitrarily small measure such that $f_n \to f$ uniformly on E^c.

Therefore, almost uniform convergence implies convergence almost everywhere. Hence, statement (ii) is true.

Also, for any $\delta > 0$, choose E such that

$$m(E) < \delta.$$

Since the convergence is uniform on E^c, for sufficiently large n,

$$|f_n(x) - f(x)| < \varepsilon$$

for all $x \in E^c$.

Hence,

$$\{x : |f_n(x) - f(x)| \geq \varepsilon\} \subseteq E,$$

so

$$m\{x : |f_n(x) - f(x)| \geq \varepsilon\} < \delta.$$

Thus, $f_n \to f$ in measure. Hence, statement (i) is true.

However, almost uniform convergence does not necessarily imply uniform convergence on the entire domain. Hence, statement (iii) is false.

Therefore, only statements (i) and (ii) are true.

Hence, option A is correct.

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46. If B is a reflexive Banach space, then its closed unit sphere S is :

(a) compact

(b) weakly compact

(c) strongly compact

(d) not compact

Answer: B. Weakly compact

Explanation:
A fundamental characterization of reflexive Banach spaces states that the closed unit ball is compact in the weak topology. Therefore, in a reflexive Banach space, the closed unit ball is weakly compact. It need not be compact in the norm topology when the space is infinite-dimensional.

47. If T is an arbitrary operator on H and if α and β are scalars such that $|\alpha| = |\beta|$ then $\alpha T + \beta T^*$ is:

(a) normal

(b) unitary

(c) adjoint

(d) self-adjoint

Answer: A. Normal

Explanation:
Let $S = \alpha T + \beta T^*$. Its adjoint is $S^* = \bar{\alpha} T^* + \bar{\beta} T$. When $|\alpha| = |\beta|$, direct multiplication shows that $SS^* = SS$. An operator satisfying this equality is called a normal operator. Therefore, $\alpha T + \beta T$ is normal.

48. An operator T on H is $____ \Leftrightarrow (T_x, x)$ is real for all x.

(a) adjoint

(b) self-adjoint

(c) normal

(d) unitary

Answer: B. Self-adjoint

Explanation:
For a bounded operator T on a complex Hilbert space, T is self-adjoint if and only if the inner product (Tx, x) is real for every x in H. A self-adjoint operator satisfies $T = T^*$. Therefore, the missing term is self-adjoint.

49. The subspace Y of a Banach space X is a Banach space if and only if :

(a) Y is open in X

(b) Y is closed in X

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(c) Y is connected subset of X

(d) Y is finite subset of X

Answer: B. Y is closed in X

Explanation:
A Banach space is a complete normed vector space. If X is a Banach space, a linear subspace Y equipped with the inherited norm is complete if and only if Y is closed in X. Therefore, Y itself is a Banach space exactly when it is a closed subspace of X.

50. The translation mapping T on the Euclidean space $(\mathbb{R}^n, d)$, $x \to a + x$ $(a \neq 0)$ has :

(a) fixed point 0

(b) fixed point 1

(c) fixed points 0 and 1

(d) no fixed points

Answer: D. No fixed points

Explanation:
The translation mapping is $T(x) = a + x$, where $a \neq 0$. A fixed point would have to satisfy $T(x) = x$. Thus,

$$a + x = x,$$

which gives $a = 0$. But the question specifically states that $a \neq 0$. Therefore, no x can satisfy the fixed-point equation, and the translation has no fixed points.

51. The Bessel equation of order α is :

(a) $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$

(b) $x^2y'' + xy' + (x^2 + \alpha^2)y = 0$

(c) $x^2y'' + xy' + (\alpha^2 - x^2)y = 0$

(d) $x^2y'' + xy' - (x^2 - \alpha^2) = 0$

Answer: A. $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$

Explanation:
The standard Bessel differential equation of order α is $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$. Its solutions are expressed using Bessel functions, usually denoted by $J_\alpha(x)$ and $Y_\alpha(x)$. Therefore, option A is correct.

52. By eliminating the arbitrary function f from $z = f(x^2 + y^2)$, the PDE is :

(a) $px = qy$

(b) $p = q$

(c) $px = q$

(d) $py = qx$

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Answer: D. $py = qx$

Explanation:
Given $z = f(x^2 + y^2)$. Differentiating partially with respect to x and y gives

$$p = \frac{\partial z}{\partial x} = 2xf'(x^2 + y^2)$$

and

$$q = \frac{\partial z}{\partial y} = 2yf'(x^2 + y^2).$$

Therefore,

$$\frac{p}{x} = \frac{q}{y},$$

which gives

$$py = qx.$$

Hence, option D is correct.

53. If $P_n(x)$ is the Legendre polynomial, then the value of $P_2(x)$ is :

(a) $\frac{3}{2}x^2$

(b) $\frac{3}{2}x - \frac{1}{2}$

(c) $\frac{3}{2}x^2 + \frac{1}{2}$

(d) $\frac{3}{2}x^2 - \frac{1}{2}$

Answer: D. $\frac{3}{2}x^2 - \frac{1}{2}$

Explanation:
The Legendre polynomials begin as $P_0(x) = 1$, $P_1(x) = x$ and $P_2(x) = \frac{1}{2}(3x^2 - 1)$. Therefore,

$$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}.$$

Hence, option D is correct.

54. The integrating factor of the differential equation

$$y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0$$

is :

(a) $\frac{1}{x^2y^3}$

(b) $\frac{1}{xy^3}$

(c) $\frac{1}{3x^2y^3}$

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(d) None of these

Answer: D. None of these

Explanation:
Given,

$$M = y(xy + 2x^2y^2) = xy^2 + 2x^2y^3$$
$$N = x(xy - x^2y^2) = x^2y - x^3y^2.$$

For an integrating factor μ, the equation

$$\mu M\,dx + \mu N\,dy = 0$$

must become exact. Therefore, it must satisfy

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x}.$$

Checking the four given options, none of them satisfies this exactness condition.

For example, for option C,

$$\mu = \frac{1}{3x^2y^3},$$

the two partial derivatives are not equal.

Therefore, the question or the given options contain an error. Option C is not mathematically correct for the differential equation as written.

55. The complementary function of

$$3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$$

is :

(a) $x[A\cos(\sqrt{2}\log x) + B\sin(\sqrt{2}\log x)]$

(b) $x[A\cos(\sqrt{2}\log x) - B\sin(\sqrt{2}\log x)]$

(c) $A\cos(\sqrt{2}\log x) + B\sin(\sqrt{2}\log x)$

(d)

$$x^{1/3}\left[A\cos\left(\frac{\sqrt{2}}{3}\log x\right) + B\sin\left(\frac{\sqrt{2}}{3}\log x\right)\right].$$

Answer: D

Explanation:
For the homogeneous equation

$$3x^2y'' + xy' + y = 0,$$

assume $y = x^m$. Then

$$y' = mx^{m-1}$$

and

$$y'' = m(m-1)x^{m-2}.$$

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Substitution gives

$$3m(m-1)+m+1=0,$$

so

$$3m^2-2m+1=0.$$

Therefore,

$$m=\frac{1\pm i\sqrt{2}}{3}.$$

Hence, the complementary function is

$$y=x^{1/3}\left[A\cos\left(\frac{\sqrt{2}}{3}\log x\right)+B\sin\left(\frac{\sqrt{2}}{3}\log x\right)\right].$$

56. Charpits method is used to solve :

(a) $F(p,q)=0$

(b) $F(p,q,x)=0$

(c) $F(x,y,z,p,q)=0$

(d) $F(z,p,q)=0$

Answer: C. $F(x,y,z,p,q)=0$

Explanation:
Charpit's method is the general method used for solving first-order nonlinear partial differential equations of the form $F(x,y,z,p,q)=0$, where $p=\dfrac{\partial z}{\partial x}$ and $q=\dfrac{\partial z}{\partial y}$. The forms shown in A, B and D are special cases of this general equation. Therefore, if the question expects the most general standard form, option C is the best answer.

57. The solution of

$$x'(t)=1+x^3(t),\quad x(0)=0$$

is :

(a) $\sec t$

(b) $\sin(t)$

(c) $\cos t$

(d) None of these

Answer: D. None of these

Explanation:
The differential equation is

$$x'(t)=1+x^3(t),\quad \text{with } x(0)=0.$$

Testing the given options shows that none satisfies the equation. For example, if $x=\tan t$, then $x'=1+\tan^2 t$, whereas the equation requires $1+\tan^3 t$. Therefore, $\tan t$ is not a solution. The actual solution is defined implicitly by integrating

$$\frac{dx}{1+x^3}=dt.$$

Hence, none of these

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58. The equation

$$u_{xx} + x^2 u_{yy} = 0$$

is :

(a) hyperbolic

(b) parabolic

(c) elliptic

(d) straight line

Answer: C. Elliptic

Explanation:
For a second-order PDE of the form

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} = 0,$$

classification depends on $B^2 - AC$.

Here,

$$A = 1, \quad B = 0 \quad \text{and} \quad C = x^2.$$

Therefore,

$$B^2 - AC = -x^2.$$

For $x \neq 0$, this is negative. Hence, the equation is elliptic. At $x = 0$, it becomes degenerate.

59. The regular singular point of the equation

$$x(x-1)\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} + y = 0$$

is :

(a) $x = 1$ and $x = 0$

(b) $y = 1$ and $y = 5$

(c) $y = 3$

(d) $y = 2$

Answer: A. $x = 1$ and $x = 0$

Explanation:
Dividing the equation by $x(x-1)$ gives

$$y'' + \left[\frac{3x-1}{x(x-1)}\right] y' + \left[\frac{1}{x(x-1)}\right] y = 0.$$

The possible singular points are $x = 0$ and $x = 1$. Applying the regular singular point conditions shows that both $x = 0$ and $x = 1$ are regular singular points.

60. The Wronskian W of the two linearly independent solutions of

$$y'' + c_1 y' + c_2 y = 0$$

satisfies the relation :

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(a) $W'' + a_1 W = 0$

(b) $W' - a_1 W = 0$

(c) $W' + a_1 W = 0$

(d) $W' - a_1 W = 0$

Answer: C. $W' + a_1 W = 0$

Explanation:
For a second-order linear differential equation

$$y'' + a_1 y' + a_2 y = 0,$$

Abel's identity gives the relation for the Wronskian W as

$$W' = -a_1 W.$$

Rearranging,

$$W' + a_1 W = 0.$$

Therefore, option C is correct.

61. If three particles are connected by rigid rods to form a triangular body with the particles at its corners. The configuration of the system is specified by giving the locations of the three particles, that is by 9 cartesian coordinates. Each rigid rod is represented Mathematically by an independent equation of constraint. Then the degrees of freedom of the system is :

(a) 3

(b) 6

(c) 9

(d) 27

Answer: B. 6

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Explanation:
The three particles require 9 Cartesian coordinates because each particle has three coordinates. The three rigid rods forming the triangle provide three independent constraint equations because the three distances between the particles remain fixed. Therefore, the number of degrees of freedom is $9 - 3 = 6$.

62. The necessary and sufficient condition for static equilibrium for an initially motionless system containing frictionless fixed constraints which may be unilateral is that the virtual work of the applied forces be :

(a) $\delta W \neq 0$

(b) $\delta W \geq 0$

(c) $\delta W \leq 0$

(d) $\delta W = 0$

Answer: C. $\delta W \leq 0$

Explanation:
For an initially motionless mechanical system with frictionless fixed constraints that may include unilateral constraints, the condition for static equilibrium is that the virtual work of the applied forces must be non-positive for every admissible virtual displacement. Therefore, $\delta W \leq 0$ is the required condition.

63. If u, v, w are functions of canonical variables and $[u, v]$ denotes the Poisson bracket of two functions u and v, then which of the following properties is false?

(a) $[u, u] = 0$

(b) $[u, v] = [v, u]$

(c) $[uv, w] = [u, w]v + u[v, w]$

(d) $[u + v, w] = [u, w] + [v, w]$

Answer: B. $[u, v] = [v, u]$

Explanation:
The Poisson bracket is antisymmetric, so

$$[u, v] = -[v, u].$$

Therefore, the statement

$$[u, v] = [v, u]$$

is false in general.

Also, by antisymmetry,

$$[u, u] = -[u, u],$$

which implies

$$[u, u] = 0.$$

The Poisson bracket also satisfies the product rule

$$[uv, w] = [u, w]v + u[v, w]$$

and linearity

$$[u + v, w] = [u, w] + [v, w].$$

Hence, only option B is false.

64. If $\{p_l, q_i\}, \{q_l, q_i\}$ are Lagrange's brackets and $[p_l, p_j], [q_l, p_j]$ are the Poisson brackets then

$$\sum_{l=1}^{n} \{p_l, q_i\}[p_l, p_j] + \sum_{l=1}^{n} \{q_l, q_i\}[q_l, p_j]$$

is equal to :

(a) δ_{ij}

(b) $-\delta_{ij}$

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(c) 0

(d) 1

Answer: C. 0

Explanation:
Lagrange brackets and Poisson brackets satisfy inverse-type relations involving the canonical variables. For the particular combination given in the question, the corresponding orthogonality relation causes the terms to cancel. Therefore,

$$\sum_{l=1}^{n}\{p_l, q_i\}[p_l, p_j] + \sum_{l=1}^{n}\{q_l, q_i\}[q_l, p_j] = 0.$$

Hence, option C is correct.

65. Consider a system of N particles whose applied forces are conservatives and are obtained from a potential energy function of the form $V(x_1, x_2, \ldots, x_{3N})$, then the virtual work dW of these applied forces is :

(a) δV

(b) $-\delta V$

(c) V

(d) $-V$

Answer: B. $-\delta V$

Explanation: For a conservative system, the applied force is obtained from the potential energy V through

$$\mathbf{F} = -\nabla V.$$

Therefore, the virtual work performed by the conservative forces is

$$\delta W = \mathbf{F} \cdot \delta \mathbf{r} = -\delta V.$$

Hence, the virtual work is equal to the negative variation of the potential energy. Therefore, option B is correct.

66. For all vectors $\vec{a}$ and $\vec{b}$, the transpose of a tensor T, denoted by T^T, is defined to be the tensor which satisfies the identity :

(a) $\vec{a} \cdot (T\vec{b}) = \vec{b} \cdot T^{\top}\vec{a}$

(b) $\vec{a} \cdot (T\vec{b}) = \vec{a} \cdot T^{\top}\vec{b}$

(c) $\vec{a} \times T\vec{b} = \vec{b} \times T^{T}\vec{a}$

(d) $\vec{a} \times T^{T}\vec{b} = T\vec{b} \times \vec{a}$

Answer: A. $\vec{a} \cdot (T\vec{b}) = \vec{b} \cdot (T^{T}\vec{a})$

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Explanation:
The transpose T^T of a tensor T is defined through the scalar-product relation

$$\vec{a} \cdot (T\vec{b}) = (T^T\vec{a}) \cdot \vec{b}.$$

Since the dot product is commutative, this can also be written as

$$\vec{a} \cdot (T\vec{b}) = \vec{b} \cdot (T^T\vec{a}).$$

Therefore, option A is correct.

67. Let $\vec{n}_1$ and $\vec{n}_2$ be two principal directions corresponding to the distinct principal values λ_1 and λ_2 respectively of a real symmetric tensor T, then :

(a) $\vec{n}_1 \cdot \vec{n}_2 = 0$

(b) $\vec{n}_1 \cdot \vec{n}_2 \neq 0$

(c) $\vec{n}_1$ & $\vec{n}_2$ are parallel and in opposite directions

(d) $\vec{n}_1$ & $\vec{n}_2$ are parallel and in same direction

Answer: A. $\vec{n}_1 \cdot \vec{n}_2 = 0$

Explanation:
For a real symmetric tensor, eigenvectors corresponding to distinct eigenvalues are orthogonal. Here, $\vec{n}_1$ and $\vec{n}_2$ are principal directions associated with distinct principal values λ_1 and λ_2. Therefore, the two principal directions are perpendicular and $\vec{n}_1 \cdot \vec{n}_2 = 0$.

68. For a symmetric stress tensor, number of independent stress components are :

(a) 3 only

(b) 6 only

(c) 9 only

(d) 2 only

Answer: B. 6 only

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Explanation:
A stress tensor in three dimensions normally contains 9 components. For a symmetric stress tensor, $T_{ij} = T_{ji}$. Therefore, the three pairs of off-diagonal components are equal, reducing the number of independent components from 9 to 6.

69. For a steady parallel, flow of an incompressible linear viscous fluid, if the flow direction is along $\hat{e}_3$, then the velocity field is of the form:

(a) $v_1 = v(x_1), v_2 = 0$ and $v_3 = 0$

(b) $v_1 = 0, v_2 = 0$ and $v_3 = v(x_3)$

(c) $v_1 = v(x_1, x_2), v_1 = 0, v_2 = 0$

(d) $v_1 = 0, v_2 = 0$ and $v_3 = v(x_1, x_2)$

Answer: D. $v_1 = 0, v_2 = 0$ and $v_3 = v(x_1, x_2)$

Explanation:
Since the parallel flow is along the $\hat{e}_3$ direction, only the third velocity component can be non-zero. In a steady fully developed parallel flow, this velocity can vary in directions perpendicular to the flow, namely x_1 and x_2. Therefore, the velocity field has the form $v_1 = 0, v_2 = 0$ and $v_3 = v(x_1, x_2)$.

70. If the tensor T is defined by the equation $T\vec{a} = k\vec{a}$, where k is a constant and $\vec{a}$ is arbitrary in terms of the identity tensor, then components of the tensor T is :

(a) $T_{ij} = k\delta_{ij}$

(b) $T_{ij} = k$

(c) $T_{ij} = \delta_{ij}$

(d) $T_{ij} = 0$

Answer: A. $T_{ij} = k\delta_{ij}$

Explanation:
The equation $T\vec{a} = k\vec{a}$ holds for every arbitrary vector $\vec{a}$. This means that T simply multiplies every vector by the scalar k. Therefore, T must be k times the identity tensor, $T = kI$. In component notation, the identity tensor has components δ_{ij}, so $T_{ij} = k\delta_{ij}$.

71. When the sample statistic lie in a certain region, then the region is called:

(a) Critical region

(b) Acceptance region

(c) Confidence region

(d) Indifference region

Answer: A. Critical region

Explanation: In hypothesis testing, the region containing values of the sample statistic for which the null hypothesis H_0 is rejected is called the critical region. It is also known as the rejection region. Therefore, option A is correct.

72. t-statistic is :

(a) $t = \dfrac{\mu - \overline{x}}{s} \times \sqrt{n}$

(b) $t = \dfrac{\overline{\mu} - \overline{x}}{\overline{s}}\sqrt{n}$

(c) $t = \dfrac{X - \mu}{s}\sqrt{n}$

(d) $t = \dfrac{\overline{X} - \mu}{s}\sqrt{n}$

Answer: D. $t = (\overline{X} - \mu)\sqrt{n}/s$

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Explanation:
For a sample of size n with sample mean $\overline{X}$ and sample standard deviation s, when the population standard deviation is unknown, the t-statistic is

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

$$= \frac{(\overline{X} - \mu)\sqrt{n}}{s}.$$

Therefore, option D is correct.

73. A random sample of size 16 has 53 as mean. The sum of squares of the deviations taken from the mean is 135. Obtain 95% confidence limits for the mean population : (Given $t_{0.05} = 2.13$)

(a) 51 to 55

(b) 55 to 59

(c) 41 to 51

(d) 31 to 41

Answer: A. 51 to 55

Explanation:
Given $n = 16$, $\overline{X} = 53$ and sum of squared deviations $= 135$.

Sample variance:

$$s^2 = \frac{135}{16 - 1} = \frac{135}{15} = 9.$$

Therefore,

$$s = 3.$$

Standard error:

$$\text{S.E.} = \frac{s}{\sqrt{n}} = \frac{3}{4} = 0.75.$$

The 95% confidence interval is

$$\overline{X} \pm t \times \text{S.E.}$$

$$= 53 \pm 2.13 \times 0.75$$

$$= 53 \pm 1.5975.$$

Therefore, the limits are approximately 51.4 to 54.6, which gives approximately 51 to 55. Hence, option A is correct.

74. The reciprocal of Standard Error (S.E) is :

(a) Measure of standard error

(b) Measure of reliability

(c) Measure of unreliability

(d) Measure of correlation

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Answer: B. Measure of reliability

Explanation:
Standard error indicates the amount of sampling variation present in an estimate. A smaller standard error indicates greater precision and reliability. Therefore, the reciprocal of standard error, 1/S.E., is regarded as a measure of reliability or precision.

75. The main assumption of F-test is :

(a) The value of each group is binomially distributed

(b) The value of each group is poissimaly distributed

(c) The value of each group is normally distributed

(d) The value of each group is exponentially distributed

Answer: C. The value of each group is normally distributed

Explanation:
The F-test assumes that the populations from which the samples are obtained are normally distributed. In applications such as ANOVA, independence of observations and appropriate assumptions about population variances are also important. Among the given choices, normal distribution is the correct answer.

76. The number of degrees of freedom for χ^2 in $m \times n$ contingency table is:

(a) $\gamma = (m-2)(n-2)$

(b) $\gamma = (m-2)(n-1)$

(c) $\gamma = (m-1)(n-2)$

(d) $\gamma = (m-1)(n-1)$

Answer: D. $\gamma = (m-1)(n-1)$

Explanation:
For a chi-square test of independence using an $m \times n$ contingency table, the degrees of freedom are calculated using

$$\text{Degrees of freedom} = (\text{Number of rows} - 1)(\text{Number of columns} - 1).$$

Therefore,

$$\gamma = (m-1)(n-1).$$

Hence, option D is correct.

77. As γ (number of degrees of freedom) tends to ∞ then χ^2-distribution becomes:

(a) Binomial distribution

(b) Normal distribution

(c) Exponential distribution

(d) Binormal distribution

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Answer: B. Normal distribution

Explanation:
The chi-square distribution is positively skewed when the degrees of freedom are small. As the number of degrees of freedom increases, the distribution becomes increasingly symmetric. For sufficiently large degrees of freedom, the chi-square distribution can be approximated by a normal distribution.

78. Given $x = 4y + 5$ and $16y = x + 4$ are the regression lines of X on Y and Y on X respectively, then $\overline{x}$ and $\overline{y}$ is :

(a) 0.75 and 8

(b) 8 and 0.75

(c) 8 and 0.50

(d) 9 and 0.75

Answer: B. 8 and 0.75

Explanation: The two regression lines intersect at the point $(\overline{x}, \overline{y})$.

The given regression lines are

$$x = 4y + 5$$

and

$$16y = x + 4.$$

Substituting $x = 4y + 5$ into the second equation, we obtain

$$16y = 4y + 5 + 4,$$
$$12y = 9,$$

and hence

$$y = \frac{9}{12} = 0.75.$$

Therefore,

$$\overline{y} = 0.75.$$

Substituting this value into $x = 4y + 5$, we get

$$x = 4(0.75) + 5 = 8.$$

Thus,

$$\overline{x} = 8$$

and

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$$\overline{y} = 0.75.$$

Hence, option B is correct.

79. If the correlation coefficient between X and Y is -1 then correlation is :

(a) negative correlation

(b) perfectly negative correlation

(c) positive correlation

(d) no correlation

Answer: B. Perfectly negative correlation

Explanation:
The correlation coefficient ranges from -1 to $+1$. A value of -1 indicates a perfect negative linear relationship between two variables. This means that when one variable increases, the other decreases in an exact linear pattern. Therefore, option B is correct.

80. The null hypothesis of one-way classification in ANOVA is :

(a) $H_0 : \mu_1 \neq \mu_2 \neq \cdots \neq \mu_n$

(b) $H_0 : \mu_1 = \mu_2, \mu_3 = \mu_4 = \cdots = \mu_n$

(c) $H_0 : \mu_1 = \mu_2, \mu_2 = \mu_3, \mu_4 = \mu_5 = \cdots = \mu_n$

(d) $H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_n$

Answer: D. $H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_n$

Explanation:
In one-way ANOVA, the null hypothesis states that all the population means are equal. ANOVA tests whether the observed differences among sample means provide sufficient evidence that at least one population mean is different. Therefore, the null hypothesis is $H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_n$.

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81. A necessary and sufficient condition that a curve be a straight line is that :

(a) $k \neq 0$ at all points

(b) $k = 0$ at all points

(c) $\vec{r}'' \neq 0$ at all points

(d) $\vec{r}' \neq \vec{a}$, as a being a constant vector

Answer: B. $k = 0$ at all points

Explanation:
The curvature k measures how much a curve bends at a particular point. A straight line does not bend anywhere, so its curvature is zero at every point. Conversely, if the curvature of a regular curve is zero everywhere, its tangent direction remains constant and the curve is a straight line.

82. The radius of spherical curvature is :

(a) $R = (\rho^2 + \sigma^2\rho'^2)^{1/2}$

(b) $R = (\rho^2 - \sigma^2\rho'^2)^{1/2}$

(c) $R = \left(\dfrac{\rho^2}{\sigma^2\rho'^2}\right)^{1/2}$

(d) $R = \left(\dfrac{\rho^2\sigma^2}{\rho'^2}\right)^{1/2}$

Answer: A. $R = (\rho^2 + \sigma^2\rho'^2)^{1/2}$

Explanation:
For a space curve, let ρ denote the radius of curvature and σ the radius of torsion. The radius of spherical curvature R is related to these quantities by

$$R^2 = \rho^2 + \sigma^2(\rho')^2.$$

Therefore,

$$R = \sqrt{\rho^2 + \sigma^2(\rho')^2}.$$

Hence, option A is correct.

83. On a given surface $\vec{r} = \vec{r}(u, v)$ where $u = u(t), v = v(t)$, then $\vec{r}$ is a function of t along the curve, and the arc length s is related to the parameter t of the metric given by :

(a) $ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2;\ E = \vec{r}_1^{\,2}, F = \vec{r}_1 \cdot \vec{r}_2, G = \vec{r}_2^{\,2}$

(b) $ds^2 = E\,du^2 - 2F\,du\,dv + G\,dv^2;\ E = \vec{r}_1^{\,2}, F = \vec{r}_2 \cdot \vec{r}_1, G = \vec{r}_2^{\,2}$

(c) $ds^2 = G\,du^2 + 2F\,du\,dv + E\,dv^2;\ E = \vec{r}_1^{\,2}, F = \vec{r}_1 \cdot \vec{r}_2, G = \vec{r}_2^{\,2}$

(d) $ds^2 = G\,du^2 - 2F\,du\,dv + E\,dv^2;\ E = \vec{r}_1^{\,2}, F = \vec{r}_1 \cdot \vec{r}_2, G = \vec{r}_2^{\,2}$

Answer: A. $ds^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2$

Explanation:
For a surface represented parametrically by $\vec{r} = \vec{r}(u, v)$, the differential displacement is

$$d\vec{r} = \vec{r}_1\,du + \vec{r}_2\,dv.$$

Taking its scalar product with itself gives

$$ds^2 = d\vec{r} \cdot d\vec{r}$$

$$= E\,du^2 + 2F\,du\,dv + G\,dv^2,$$

where

$$E = \vec{r}_1 \cdot \vec{r}_1,$$

$$F = \vec{r}_1 \cdot \vec{r}_2,$$

$$G = \vec{r}_2 \cdot \vec{r}_2.$$

This expression is called the first fundamental form or metric of the surface.

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84. On a right circular cone of semi-vertical angle α, every point can be joined to itself by a geodesic arc if $\alpha < \pi/6$. If this condition is satisfied, then the number of geodesic arcs joining a point to itself is the greatest integer less than :

(a) $2\sin^{-1}\alpha$

(b) $2\sin\alpha$

(c) $2\cos\alpha$

(d) $(2\sin\alpha)^{-1}$

Answer: D. $(2\sin\alpha)^{-1}$

Explanation:
For a right circular cone with semi-vertical angle α, the number of geodesic arcs that can join a point back to itself depends on the geometry obtained by developing the cone onto a plane. When $\alpha < \pi/6$, the number is determined by the greatest integer less than $1/(2\sin\alpha)$. Therefore, option D is correct.

85. If A is the area of a geodesic disk of centre P and radius r, then which of the following is true ?

(a) $(K)_P = \lim\limits_{r\to\infty} \dfrac{\pi r^2 - A}{\frac{1}{2}\pi r^4}$

(b) $(K)_P = \lim\limits_{r\to 0} \dfrac{\pi r^2 - A}{\frac{1}{12}\pi r^4}$

(c) $(K)_P = \lim\limits_{r\to\infty} \dfrac{\pi r^2 + A}{\frac{1}{3}\pi r^4}$

(d) $(K)_P = \lim\limits_{r\to 0} \dfrac{\pi r^2 - A}{\frac{1}{4}\pi r^4}$

Answer: B. $K_P = \lim\limits_{r\to 0} \dfrac{\pi r^2 - A}{(1/12)\pi r^4}$

Explanation:
For a sufficiently small geodesic disk of radius r centred at P, its area has the expansion

$$A = \pi r^2 - \frac{K_P}{12}\pi r^4 + \text{higher-order terms},$$

where K_P is the Gaussian curvature at P. Rearranging and taking the limit as r approaches zero gives

$$K_P = \lim_{r\to 0} \frac{\pi r^2 - A}{(1/12)\pi r^4}.$$

Hence, option B is correct.

86. If G is a simple, connected and non-complete graph, then which of the following is necessarily true?

(a) G has three vertices u, v and w such that $uv, vw, uw \in E(G)$

(b) G has three vertices u, v and w such that $uv, vw \notin E(G)$ and $uw \in E(G)$

(c) G has three vertices u, v and w such that $uv, vw \in E(G)$ and $uw \notin E(G)$

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(d) G has three vertices u, v and w such that $uv, vw, uw \notin E(G)$

Answer: C. $uv, vw \in E(G)$ and $uw \notin E(G)$

Explanation: Since G is non-complete, there exist two distinct non-adjacent vertices x and y. Since G is connected, there is a path joining x and y.

Consider a shortest path

$$x = v_0, v_1, v_2, \ldots, v_k = y$$

joining them. Since x and y are not adjacent, $k \geq 2$.

Let

$$u = v_0, \qquad v = v_1, \qquad w = v_2.$$

Since these are three consecutive vertices on the path,

$$uv \in E(G)$$

and

$$vw \in E(G).$$

If $uw \in E(G)$, then the two edges uv and vw could be replaced by the single edge uw, producing a shorter path from x to y. This contradicts the choice of the original path as a shortest path.

Therefore,

$$uv, vw \in E(G)$$

and

$$uw \notin E(G).$$

Hence, option C is correct.

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87. How many number of distinct spanning trees of K_6 ?

(a) 729

(b) 4096

(c) 1296

(d) 216

Answer: C. 1296

Explanation:
Cayley's formula states that the number of distinct spanning trees of the complete graph K_n is

$$n^{n-2}.$$

For K_6,

$$\text{Number of spanning trees} = 6^{6-2}$$

$$= 6^4$$

$$= 1296.$$

Therefore, option C is correct.

88. If $r(k, l)$ are the Ramsey numbers, for any positive integers k and l. Then $r(3, 7)$ is equal to :

(a) 14

(b) 18

(c) 23

(d) 9

Answer: C. 23

Explanation:
The Ramsey number $r(3, 7)$ is the smallest integer n such that every graph on n vertices contains either a complete subgraph K_3 or its complement contains K_7. The known value is

$$r(3, 7) = 23.$$

Therefore, option C is correct.

89. If $\pi_k(G)$ is the number of distinct K-colourings of any graph G and G is simple, then for any edge e of G, $\pi_k(G) =$

(a) $\pi_k(G + e) - \pi_k(G \cdot e)$

(b) $\pi_k(G + e)$

(c) $\pi_k(G - e) - \pi_k(G \cdot e)$

(d) $\pi_k(G \cdot e)$

Answer: C. $\pi_k(G) = \pi_k(G - e) - \pi_k(G \cdot e)$

Explanation:
The chromatic polynomial satisfies the deletion-contraction relation. For an edge e of a graph G,

$$\pi_k(G) = \pi_k(G - e) - \pi_k(G \cdot e),$$

where $G - e$ means deleting the edge e and $G \cdot e$ means contracting the edge e. Therefore, option C is correct.

90. Let G be a connected planar graph with 6 vertices and 8 edges. Then the number of regions are :

(a) 6

(b) 8

(c) 2

(d) 4

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Answer: D. 4

Explanation:
For a connected planar graph, Euler's formula is

$$V - E + F = 2,$$

where V is the number of vertices, E is the number of edges and F is the number of regions.
Given $V = 6$ and $E = 8$,

$$6 - 8 + F = 2.$$

Therefore,

$$F = 4.$$

Hence, the graph has 4 regions and option D is correct.

91. For a viscous fluid in contact with a stationary rigid boundary, if $\vec{q}$ denotes the fluid velocity at a point P on the boundary, then the required boundary condition is:

(a) $\vec{q} \cdot \vec{n} = 0$

(b) $\vec{q} \times \vec{n} = 0$

(c) $\vec{q} = \vec{0}$

(d) $\vec{n} = \vec{0}$

Answer: C. $\vec{q} = \vec{0}$

Explanation:

For a viscous fluid in contact with a solid boundary, the no-slip condition states that the velocity of the fluid at the boundary is equal to the velocity of the boundary.
For a stationary rigid boundary, the boundary velocity is zero. Hence,

$$\vec{q} = \vec{0}.$$

This condition includes both the no-penetration condition and the absence of tangential slip.
Therefore, option C is correct.

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92. If $\vec{q}$ is the velocity field of a fluid, then the vorticity vector is defined as:

(a) $\nabla^2 \vec{q}$

(b) $\frac{1}{2} \nabla \times \vec{q}$

(c) $\nabla \cdot \vec{q}$

(d) $\nabla \times \vec{q}$

Answer: D. $\nabla \times \vec{q}$

Explanation:

The vorticity vector of a fluid velocity field $\vec{q}$ is defined by

$$\vec{\omega} = \nabla \times \vec{q}.$$

It measures the local rotational tendency of the fluid.

The quantity

$$\frac{1}{2}\nabla \times \vec{q}$$

represents the local angular velocity or spin vector of a fluid element.

Hence, the vorticity vector is

$$\boxed{\nabla \times \vec{q}}.$$

Therefore, option D is correct.

93. If C is a closed curve drawn in a moving fluid and $\vec{q}$ denotes the velocity of the fluid, then the circulation Γ around C is:

(a) $\Gamma = \oint_C \vec{q} \cdot d\vec{r}$

(b) $\Gamma = \oint_C \nabla \cdot \vec{q}\, ds$

(c) $\Gamma = \oint_C \vec{q} \cdot \vec{n}\, ds$

(d) $\Gamma = \oint_C \nabla^2 \vec{q}\, ds$

Answer: A. $\Gamma = \oint_C \vec{q} \cdot d\vec{r}$

Explanation:

Circulation is defined as the line integral of the tangential component of the fluid velocity around a closed curve.

Therefore,

$$\Gamma = \oint_C \vec{q} \cdot d\vec{r}.$$

By Stokes' theorem, it may also be written as

$$\Gamma = \iint_S (\nabla \times \vec{q}) \cdot \vec{n}\, dS,$$

where S is any surface bounded by C.

Hence, option A is correct.

94. A conical pipe has diameters of 10 cm and 15 cm at the two ends. If the velocity at the smaller end is 2 m/sec, then the velocity at the other end is :

(a) 0.84 m/sec

(b) 0.89 m/sec

(c) 0.82 m/sec

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(d) 0.80 m/sec

Answer: B. 0.89 m/sec

Explanation:
For incompressible steady flow, the continuity equation gives

$$A_1 v_1 = A_2 v_2.$$

Since area is proportional to the square of diameter,

$$v_2 = v_1 \left(\frac{d_1}{d_2}\right)^2$$

$$= 2\left(\frac{10}{15}\right)^2$$

$$= 2 \times \frac{100}{225}$$

$$= 0.889 \text{ m/sec.}$$

Therefore, the velocity at the larger end is approximately 0.89 m/sec.

95. The relation between the velocity potential ϕ and the stream function Ψ is :

(a) $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ and $\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$

(b) $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ and $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

(c) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$ and $\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$

(d) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$ and $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

Answer: B. $\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$ and $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

Explanation:
For a two-dimensional potential flow,

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

and

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}.$$

These relations are analogous to the Cauchy-Riemann equations and connect the velocity potential ϕ with the stream function ψ. Therefore, option B is correct.

96. Which of the following statements about conformal transformations is not correct?

(a) If both the magnitude and sense of an angle are preserved under a transformation, then the transformation is conformal.

(b) A map $w = f(z)$ is conformal at a point where f is analytic and $f'(z) \neq 0$.

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(c) In the conformal representation of two-dimensional fluid motion, a source is transformed into a source at the corresponding point.

(d) Under every conformal transformation, a doublet is transformed into a doublet of the same strength.

Answer: D. Under every conformal transformation, a doublet is transformed into a doublet of the same strength.

Explanation:
A conformal transformation preserves the magnitude and orientation of angles locally.

If

$$w = f(z)$$

is analytic and

$$f'(z) \neq 0,$$

then the transformation is conformal at that point.

Sources are preserved as sources under conformal mapping.

However, although a doublet is transformed into a doublet, its strength need not remain unchanged. The transformed strength generally depends on the derivative of the conformal transformation at the corresponding point.

Therefore, option D is not correct.

97. The ratio

$$\frac{\text{inertial force}}{\text{viscous force}}$$

is said to be :

(a) Magnet number

(b) Reynold's number

(c) Kirchoff's number

(d) Picket number

Answer: B. Reynolds number

Explanation:
Reynolds number is a dimensionless quantity representing the ratio of inertial forces to viscous forces in fluid flow. It is commonly expressed as

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}.$$

It is important for determining whether a flow tends to be laminar or turbulent. Therefore, option B is correct.

98. Which of the following statement is not true about linear programming problem ?

(a) All constraints must be linear relationships

(b) Objective function must be linear

(c) All the constraints and decision variables must be of either '$\leq$' or '$\geq$' type

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(d) All decision variables must be non-negative

Answer: C

Explanation:
In a linear programming problem, the objective function and constraints must be linear, and decision variables are normally subject to non-negativity conditions. However, constraints are not restricted to only two inequality types. They may involve $\leq$, $\geq$ or $=$ relationships. Therefore, the statement in option C is not correct.

99. Which of the following statement is not correct ?

(a) Feasible solution of an Linear Programming Problem is independent of the objective function.

(b) The feasible region of an Linear Programming Problem must be a convex set.

(c) The feasible region for a constraint is restricted if its '$\geq$' or '$\leq$' sign is replaced by a '$=$' sign

(d) It is not possible to obtain feasible solution of an Linear Programming Problem by graphical method.

Answer: D. It is not possible to obtain a feasible solution by graphical method

Explanation:
This statement is false because the graphical method can be used to determine the feasible region and optimal solution of a linear programming problem involving two decision variables. The feasible region is obtained by plotting the constraints and identifying their common region. Therefore, option D is the incorrect statement.

100. Which of the following statements about a transportation problem is not correct?

(a) If certain routes are prohibited, their corresponding cost elements may be assigned a very large value M.

(b) The number of cells involved in a closed loop in a transportation table is always even.

(c) If all transportation costs c_{ij} are multiplied by a positive constant k, then the minimum transportation cost is also multiplied by k.

(d) If $u_i + v_j - c_{ij} = 0$ for an occupied cell in an optimal solution, then the transportation problem necessarily has multiple optimal solutions.

Answer: D

Explanation:

In the MODI method, for every occupied basic cell,

$$u_i + v_j = c_{ij}.$$

Therefore,

$$u_i + v_j - c_{ij} = 0$$

holds naturally for occupied cells and does not indicate the existence of an alternative optimal solution.

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Multiple optimal solutions occur when an appropriate **unoccupied** cell has zero opportunity cost at the optimum.

Also, a prohibited route may be assigned a very large cost M, and every closed transportation loop contains an even number of cells.

Multiplying all costs by a positive constant multiplies the optimal transportation cost by the same constant.

Hence, option D is the statement that is not correct.

101. In 1855, Lord Dalhousie stopped the pension and confiscated the jagirs of _____ on the death of the Rajah.

(a) Senji

(b) Tanjore

(c) Palayankottai

(d) Ramnad

Answer: B. Tanjore

Explanation:
After the death of the Raja of Tanjore, Lord Dalhousie applied his annexation policy and ended certain privileges associated with the royal family. In 1855, the pension was stopped and the jagirs connected with Tanjore were confiscated. Therefore, Tanjore is the correct answer.

102. A, B and C can complete a piece of work in 10, 15 and 18 days respectively. In how many days would all of them complete the same work working together ?

(a) $4\frac{1}{2}$ days

(b) $3\frac{1}{2}$ days

(c) $5\frac{1}{2}$ days

(d) $2\frac{1}{2}$ days

Answer: A. $4\frac{1}{2}$ days

Explanation:
A completes the work in 10 days, so A's one-day work is $\frac{1}{10}$. Similarly, B's one-day work is $\frac{1}{15}$ and C's is $\frac{1}{18}$.
Together,

$$\frac{1}{10} + \frac{1}{15} + \frac{1}{18}$$
$$= \frac{9}{90} + \frac{6}{90} + \frac{5}{90}$$
$$= \frac{20}{90}$$

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$$= \frac{2}{9}.$$

Therefore, the time required is

$$1 \div \left(\frac{2}{9}\right) = \frac{9}{2} = 4.5 \text{ days.}$$

Hence, they complete the work in $4\frac{1}{2}$ days.

103. Who among the following was the organiser of Dindugul Confederacy against British ?

(a) Marudhu Pandyan

(b) Pulithevan

(c) Gopala Nayak

(d) Keralavarma

Answer: C. Gopala Nayak

Explanation:
The supplied answer key appears incorrect. Gopala Nayak, also known as Gopal Nayak of Virupakshi, played a leading role in organising the Dindigul Confederacy against British rule. He coordinated resistance with other South Indian leaders. Therefore, the correct answer is C. Gopala Nayak, not Pulithevan.

104. Who shifted his capital from Uraiyur to Kaveripattanam ?

(a) Cheran Chenkuttuvan

(b) Karikalan

(c) Nedunchezhiyan

(d) Rajaraja I

Answer: B. Karikalan

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Explanation:
Karikala Chola was one of the most prominent rulers of the Early Cholas. He is traditionally associated with shifting the Chola capital from Uraiyur to Kaveripattinam, also known as Puhar. Kaveripattinam subsequently developed into an important port and commercial centre.

105. Which of the following temple is an example of the early temple architecture of the Cholas ?

(a) Tanjore Temple

(b) Koranganatha Temple

(c) Brahadisvara Temple

(d) Subramaniya Temple

Answer: B. Koranganatha Temple

Explanation:
The Koranganatha Temple at Srinivasanallur is an important example of early Chola temple architecture. It represents the architectural development that preceded the massive temples constructed during the later imperial Chola period. Therefore, option B is correct.

106. The vector of Malarian parasite is :

(a) Female Aedes mosquitoes

(b) Female Anopheles mosquitoes

(c) Female Culex mosquitoes

(d) Plasmodium malariae

Answer: B. Female Anopheles mosquitoes

Explanation:
Malaria is caused by Plasmodium parasites and is transmitted to humans through the bite of infected female Anopheles mosquitoes. The mosquito acts as the vector carrying the parasite from one person to another. Therefore, option B is correct.

107. Where was the first experimental electric telegraph line opened in India?

(a) Bombay to Calcutta

(b) Calcutta to Delhi

(c) Calcutta to Diamond Harbour

(d) Calcutta to Agra

Answer: C. Calcutta to Diamond Harbour

Explanation:
The first experimental electric telegraph line in India was established between Calcutta and Diamond Harbour.

The line was developed under the supervision of Dr. William O'Shaughnessy and became operational in the early development of telegraph communication in India.

Therefore, the correct route was

Calcutta to Diamond Harbour.

Hence, option C is correct.

108. Who was the editor of the Newspaper "Tamil Nadu"?

(a) E.V. Ramasami Naicker

(b) Varadharajulu Naidu

(c) George Joseph

(d) A. Gopalakrishnan

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Answer: B. Varadharajulu Naidu

Explanation:
Dr. P. Varadarajulu Naidu was a prominent nationalist, journalist and political activist in the Madras Presidency. He was associated with the Tamil newspaper "Tamil Nadu" and served as its editor. Therefore, option B is correct.

109. Who among the following constructed Gangaikonda Cholapuram ?

(a) Rajaraja

(b) Rajendra

(c) Karikalan

(d) Parantaka

Answer: B. Rajendra Chola I

Explanation:
Rajendra Chola I founded Gangaikonda Cholapuram after his successful northern military campaign toward the Ganges. He established it as the new Chola capital and constructed the famous Gangaikondacholisvaram temple there. Therefore, Rajendra Chola I is the correct answer.

110. 'Silver fir' trees are widely found in :

(a) Eastern Himalayas

(b) Eastern Ghats

(c) Western Ghats

(d) Ganga Delta

Answer: A. Eastern Himalayas

Explanation:
Silver fir is a characteristic coniferous tree found in the high-altitude Himalayan region. In India, silver fir forests are widely associated with the temperate and alpine vegetation of the Himalayas, including the Eastern Himalayas. Therefore, among the given options, Eastern Himalayas is correct.

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PART - B

1. Let

$$f(x) = \begin{cases} \dfrac{|x-3|}{x-3}, & x \neq 3 \\ 0, & x = 3 \end{cases}$$

Then $\lim\limits_{x \to 3} f(x)$:

(a) is 1

(b) is -1

(c) is 0

(d) does not exists

Answer: D. Does not exist

Explanation:
For $x \neq 3$,

$$f(x) = \frac{|x-3|}{x-3}.$$

We have to examine the limit from both sides of $x = 3$.

When $x > 3$, $x - 3$ is positive. Therefore,

$$|x-3| = x-3,$$

and hence

$$f(x) = \frac{x-3}{x-3} = 1.$$

Thus,

$$\text{Right-hand limit} = 1.$$

When $x < 3$, $x - 3$ is negative. Therefore,

$$|x-3| = -(x-3),$$

and hence

$$f(x) = \frac{-(x-3)}{x-3} = -1.$$

Thus,

$$\text{Left-hand limit} = -1.$$

Since the left-hand limit and right-hand limit are different, the limit does not exist.

The value $f(3) = 0$ does not affect this conclusion because a limit depends on the values of the function near the point, not necessarily at the point itself.

Therefore, option D is correct.

2. The value of

$$\left| \int_0^1 \frac{\cos nx}{1+x}\, dx \right|$$

is :

(a) $\leq \ln 2$

(b) $\leq \ln 3$

(c) ≤ 2

(d) ≤ 3

Answer: A. $\leq \ln 2$

Explanation:
We are given

$$\left| \int_0^1 \frac{\cos nx}{1+x}\, dx \right|.$$

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Using the standard inequality

$$\left|\int f(x)\,dx\right| \le \int |f(x)|\,dx,$$

we get

$$\left|\int_0^1 \frac{\cos nx}{1+x}\,dx\right| \le \int_0^1 \frac{|\cos nx|}{1+x}\,dx.$$

Since

$$|\cos nx| \le 1,$$

we have

$$\le \int_0^1 \frac{1}{1+x}\,dx.$$

Now,

$$\int_0^1 \frac{1}{1+x}\,dx = [\ln(1+x)]_0^1 = \ln 2 - \ln 1 = \ln 2.$$

Therefore,

$$\left|\int_0^1 \frac{\cos nx}{1+x}\,dx\right| \le \ln 2.$$

Hence, option A is correct.

3. The value of

$$\int_0^\infty \sqrt{y}\, e^{-y^3}\,dy$$

is :

(a) $\frac{2\sqrt{\pi}}{5}$

(b) $\sqrt{\pi}$

(c) $\frac{\sqrt{\pi}}{3}$

(d) $\frac{2\sqrt{\pi}}{3}$

Answer: C. $\frac{\sqrt{\pi}}{3}$

Explanation:

Consider

$$I = \int_0^\infty \sqrt{y}\, e^{-y^3}\,dy.$$

Write $\sqrt{y}$ as $y^{1/2}$:

$$I = \int_0^\infty y^{1/2} e^{-y^3}\,dy.$$

Use the substitution

$$t = y^3.$$

Then,

$$y = t^{1/3}$$

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and

$$dy = \frac{1}{3}t^{-2/3}\,dt.$$

Also,

$$y^{1/2} = t^{1/6}.$$

Therefore,

$$I = \frac{1}{3}\int_0^\infty t^{1/6}t^{-2/3}e^{-t}\,dt$$

$$= \frac{1}{3}\int_0^\infty t^{-1/2}e^{-t}\,dt.$$

Using the Gamma function,

$$\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}\,dt.$$

Here,

$$a - 1 = -\frac{1}{2},$$

so $a = \frac{1}{2}$.

Therefore,

$$I = \frac{1}{3}\Gamma\left(\frac{1}{2}\right).$$

We know that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

Hence,

$$I = \frac{\sqrt{\pi}}{3}.$$

Therefore, option C is correct.

4. $L(x) = \int_1^x \frac{dt}{t}\,(x > 0)$ is called the natural logarithm of x denoted by $\log x$. The function L does not satisfy the condition :

(a) $L : (0, \infty) \to \mathbb{R}$ is bijective

(b) $L(x) > 0$ for $x > 0$

(c) $L(xy) = L(x) + L(y)$ for x, y in $(0, \infty)$

(d) $L(x^a) = aL(x)$ for $x > 0$ & $a \in \mathbb{R}$

Answer: B. $L(x) > 0$ for $x > 0$

Explanation:

The natural logarithm is defined by

$$L(x) = \int_1^x \frac{dt}{t}, \quad \text{for } x > 0.$$

The statement $L(x) > 0$ for every $x > 0$ is false.

If $x > 1$, then

$$L(x) > 0.$$

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If $x = 1$,

$$L(1) = 0.$$

But if

$$0 < x < 1,$$

then

$$L(x) < 0.$$

For example,

$$L\left(\frac{1}{2}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2 < 0.$$

Therefore, it is incorrect to say that $L(x)$ is positive for every positive x.

The other properties are standard logarithmic properties:

$$L(xy) = L(x) + L(y),$$

and

$$L(x^a) = aL(x).$$

Also, $\ln x$ maps $(0, \infty)$ bijectively onto $\mathbb{R}$.

Therefore, option B is the condition that L does not satisfy.

5. For which function $f(z)$ given below $f'(z)$ does not exists in the entire complex plane ?

(a) $f(z) = \overline{z}^2$

(b) $f(z) = e^z$

(c) $f(z) = \dfrac{1}{z}$

(d) $f(z) = z - \overline{z}$

Answer: A and D

Explanation:

For a complex function to possess a complex derivative throughout the complex plane, it must satisfy the Cauchy-Riemann conditions at every point.

For option A,

$$f(z) = \overline{z}^2.$$

Functions involving the complex conjugate $\overline{z}$ generally fail the Cauchy-Riemann conditions. In fact, $\overline{z}^2$ is complex differentiable only at $z = 0$, not throughout the complex plane. Therefore, its derivative does not exist on the entire complex plane.

For option D,

$$f(z) = z - \overline{z}.$$

Let $z = x + iy$. Then

$$z - \overline{z} = (x + iy) - (x - iy) = 2iy.$$

Thus,

$$u(x, y) = 0,$$

$$v(x, y) = 2y.$$

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The Cauchy-Riemann equations require

$$u_x = v_y$$

and

$$u_y = -v_x.$$

But

$$u_x = 0 \quad \text{and} \quad v_y = 2.$$

Since $0 \neq 2$, the Cauchy-Riemann equations fail everywhere. Hence, f is nowhere complex differentiable.

Option B, e^z, is entire.

Option C, $\dfrac{1}{z}$, is differentiable everywhere except $z = 0$.

Therefore, according to the wording "does not exist in the entire complex plane," both A and D are appropriate, matching the supplied key.

6. The circle of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k2^k}(z-1-i)^k$$

is :

(a) $|z-1-i| = \dfrac{1}{2}$

(b) $|z-1-i| = 1$

(c) $|z-1-i| = 2$

(d) $|z-1-i| = 4$

Answer: C. $|z-1-i| = 2$

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Explanation:
The power series is centred at

$$z_0 = 1 + i.$$

Its general coefficient is

$$a_k = \frac{(-1)^k}{k2^k}.$$

To determine the radius of convergence R, use the ratio test.

Consider

$$\left|\frac{a_k}{a_{k+1}}\right| = \left|\frac{(-1)^k}{k2^k} \times \frac{(k+1)2^{k+1}}{(-1)^{k+1}}\right|.$$

Taking absolute values,

$$= \frac{2(k+1)}{k}.$$

As k approaches infinity,

$$\frac{2(k+1)}{k} \to 2.$$

Therefore,

$$R = 2.$$

Thus, the series converges for

$$|z-(1+i)|<2.$$

The corresponding circle of convergence has radius 2:

$$|z-1-i|=2.$$

Hence, option C is correct.

7. The function $u(x,y)=4x^2-4y^2+3y$ is harmonic and its harmonic conjugate is (k - constant) :

(a) $v(x,y)=x-4xy+k$

(b) $v(x,y)=8xy-3x+k$

(c) $v(x,y)=5x^2-3y+k$

(d) $v(x,y)=8xy-3y+k$

Answer: B. $v(x,y)=8xy-3x+k$

Explanation:

Given

$$u(x,y)=4x^2-4y^2+3y.$$

For $u+iv$ to be analytic, the Cauchy-Riemann equations must hold:

$$u_x=v_y$$

and

$$u_y=-v_x.$$

First,

$$u_x=8x.$$

Therefore,

$$v_y=8x.$$

Integrating with respect to y,

$$v=8xy+\phi(x),$$

where $\phi(x)$ is a function of x.

Now differentiate u with respect to y:

$$u_y=-8y+3.$$

Using

$$u_y=-v_x,$$

we obtain

$$v_x=8y-3.$$

But from

$$v=8xy+\phi(x),$$

we get

$$v_x=8y+\phi'(x).$$

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Therefore,

$$8y + \phi'(x) = 8y - 3.$$

Hence,

$$\phi'(x) = -3.$$

Integrating,

$$\phi(x) = -3x + k.$$

Therefore,

$$v(x, y) = 8xy - 3x + k.$$

Hence, option B is correct.

8. Which one of the following functions $f(z)$ obey reflection principle with respect to the domain $|z - 2| < 1$?

(a) $f(z) = z + 1$

(b) $f(z) = iz^2$

(c) $f(z) = z + i$

(d) $f(z) = z - i$

Answer: A. $f(z) = z + 1$

Explanation:

The Schwarz reflection principle applies when an analytic function has suitable real boundary values on a domain symmetric with respect to the relevant reflection.

The domain

$$|z - 2| < 1$$

is a circle centred at the real point 2, so it is symmetric about the real axis.

For option A,

$$f(z) = z + 1.$$

Its coefficients are real, and therefore it satisfies

$$f(\overline{z}) = \overline{f(z)}.$$

Indeed,

$$f(\overline{z}) = \overline{z} + 1$$

and

$$\overline{f(z)} = \overline{z + 1} = \overline{z} + 1.$$

Thus, the required symmetry is satisfied.

Functions involving the additional imaginary constants in the other options do not have the same real-axis reflection property.

Therefore, option A is correct.

9. The number of non-isomorphic abelian group of order 108 is :

(a) 6

(b) 9

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(c) 5

(d) 12

Answer: A. 6

Explanation:

We need the number of non-isomorphic abelian groups of order 108.

First factorize:

$$108 = 4 \times 27$$
$$= 2^2 \times 3^3.$$

According to the Fundamental Theorem of Finite Abelian Groups, the number of non-isomorphic abelian groups of order p^n equals the number of partitions of n.

For 2^2, the exponent is 2.

The partitions of 2 are:

$$2$$
$$1+1.$$

Therefore, there are 2 possibilities for the 2-primary component.

For 3^3, the exponent is 3.

The partitions of 3 are:

$$3$$
$$2+1$$
$$1+1+1.$$

Therefore, there are 3 possibilities for the 3-primary component.

Hence, the total number of non-isomorphic abelian groups is

$$2 \times 3 = 6.$$

Therefore, option A is correct.

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10. The matrix

$$A = \begin{pmatrix} \frac{1+i}{\sqrt{7}} & \frac{2+i}{\sqrt{7}} \\ \frac{2-i}{\sqrt{7}} & \frac{-1+i}{\sqrt{7}} \end{pmatrix}$$

is :

(a) Orthogonal matrix

(b) Skew symmetric matrix

(c) Unitary matrix

(d) Skew Hermitian matrix

Answer: C. Unitary matrix

Explanation:
A complex square matrix A is called unitary if

$$A^*A = I,$$

where A^* denotes the conjugate transpose of A.

The given matrix is

$$A = \frac{1}{\sqrt{7}} \begin{pmatrix} 1+i & 2+i \\ 2-i & -1+i \end{pmatrix}.$$

Consider its columns.

The squared norm of the first column is

$$|1+i|^2 + |2-i|^2$$

$$= (1^2+1^2) + (2^2+1^2)$$

$$= 2+5$$

$$= 7.$$

Because of the factor $1/\sqrt{7}$, its normalized squared length becomes

$$\frac{7}{7} = 1.$$

Similarly, the squared norm of the second column is

$$|2+i|^2 + |-1+i|^2$$

$$= 5+2$$

$$= 7,$$

which also becomes 1 after multiplication by $1/\sqrt{7}$.

The inner product of the two columns is zero, so the columns are mutually orthogonal.

Thus, the columns form an orthonormal set and

$$A^*A = I.$$

Therefore, the matrix is unitary and option C is correct.

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11. Let $T : P_3(R) \to P_2(R)$ [Here $P_n(F)$ is set of all polynomials of degree $\leq n$] be a linear transformation defined by $T(f(x)) = \frac{df}{dx}$. Let β and γ be the standard basis for $P_3(R)$ and $P_2(R)$ respectively. Then the corresponding matrix $[T]_\beta^\gamma$ is _____ .

(a) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$

Answer: D

Explanation:

The standard basis for $P_3(R)$ is

$$\beta = \{1, x, x^2, x^3\},$$

and the standard basis for $P_2(R)$ is

$$\gamma = \{1, x, x^2\}.$$

The transformation is differentiation:

$$T(f(x)) = \frac{df}{dx}.$$

Apply T to each basis element of $P_3(R)$:

$$T(1) = 0$$

$$T(x) = 1$$

$$T(x^2) = 2x$$

$$T(x^3) = 3x^2.$$

Now express each result using the basis $\gamma = \{1, x, x^2\}$:

$$T(1) = (0, 0, 0)$$

$$T(x) = (1, 0, 0)$$

$$T(x^2) = (0, 2, 0)$$

$$T(x^3) = (0, 0, 3).$$

These coordinate vectors form the columns of the matrix. Therefore,

$$[T] = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}.$$

Hence, option D is correct.

12. The Linear Transformation determined by the following matrix $T : R^2 \to R^2$ given by

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

with respect to the standard basis is :

(a) $T(a, b) = (\cos^2\theta + \sin\theta, a\sin\theta + b\cos\theta)$

(b) $T(a, b) = (a\cos\theta - b\sin\theta,\ a\sin\theta + b\cos\theta)$

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(c) $T(a,b) = (a\cos\theta - b\sin\theta, a\sin\theta - b\cos\theta)$

(d) $T(a,b) = (a\cos\theta + b\sin\theta, a\sin\theta + b\cos\theta)$

Answer: B

Explanation:
The given matrix is

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

For the vector (a,b), matrix multiplication gives

$$T(a,b) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}.$$

Therefore, the first component is

$$a\cos\theta - b\sin\theta,$$

and the second component is

$$a\sin\theta + b\cos\theta.$$

Hence,

$$T(a,b) = (a\cos\theta - b\sin\theta, a\sin\theta + b\cos\theta).$$

This is the standard transformation representing a counterclockwise rotation through an angle θ.

13. Let $f : (-1,1) \to R$ be defined by

$$f(x) = \frac{x}{1-x^2}.$$

Then f is a _____ .

(a) bijection

(b) continuous function

(c) one to one function

(d) homeomorphism

Answer: All the options are correct

Explanation:
The function is

$$f(x) = \frac{x}{1-x^2},$$

with

$$-1 < x < 1.$$

First, f is continuous because both numerator and denominator are continuous and

$$1 - x^2 \neq 0$$

for every x in $(-1,1)$.

Now differentiate:

$$f'(x) = \frac{(1-x^2) + 2x^2}{(1-x^2)^2} = \frac{1+x^2}{(1-x^2)^2}.$$

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For

$$-1 < x < 1,$$

$$f'(x) > 0.$$

Therefore, f is strictly increasing and hence one-to-one.

Also,

$$\lim_{x \to -1^+} f(x) = -\infty$$

and

$$\lim_{x \to 1^-} f(x) = +\infty.$$

Since f is continuous and strictly increasing from $-\infty$ to $+\infty$, its range is the whole of R. Therefore, it is onto R.

Hence, f is bijective.

Finally, a continuous strictly monotonic bijection from an interval onto an interval has a continuous inverse. Therefore, f is a homeomorphism from $(-1, 1)$ onto R.

Thus, f is continuous, one-to-one, bijective and a homeomorphism. Hence, all the options are correct.

14. Let X and Y be topological spaces. Let $f : X \to Y$ be a function.

(A) : f is continuous.

(B) : for every closed set B of Y, then set $f^{-1}(B)$ is closed in X.

Then which of the following gives the complete relationship between (A) and (B)?

(a) $(A) \Rightarrow (B)$

(b) $(B) \Rightarrow (A)$

(c) $(A) \Leftrightarrow (B)$

(d) (A) and (B) are not related

Answer: C. $(A) \Leftrightarrow (B)$

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Explanation:

One of the fundamental characterizations of continuity in topology states that a function

$$f : X \to Y$$

is continuous if and only if the inverse image of every closed set in Y is closed in X.

Thus, if f is continuous and B is closed in Y, then

$$f^{-1}(B)$$

is closed in X.

Therefore,

$$(A) \Rightarrow (B).$$

Conversely, suppose the inverse image of every closed subset of Y is closed in X. If U is open in Y, then $Y - U$ is closed.

Therefore,

$$f^{-1}(Y - U)$$

is closed in X.

But

$$f^{-1}(Y - U) = X - f^{-1}(U).$$

Thus, $f^{-1}(U)$ is open in X. Hence, f is continuous.

Therefore,

$$(B) \Rightarrow (A).$$

So,

$$(A) \Leftrightarrow (B).$$

Options A and B individually state true implications, but option C gives the complete equivalence and is therefore the best answer.

15. Let A and B be connected sets in a space X. Then which of the following is true ?

(a) $A \cup B$ is connected

(b) $A \cup B$ is disconnected

(c) $A \cup B$ is connected, if $A \cap B \neq \phi$

(d) $A \cup B$ is connected, if $A \cap B = \phi$

Answer: C. $A \cup B$ is connected if $A \cap B \neq \emptyset$

Explanation:

A standard theorem in topology states that if two connected subsets A and B have at least one common point, then their union is also connected.

That is, if

$$A \cap B \neq \emptyset,$$

then

$$A \cup B$$

is connected.

The reason is that the common point connects the two sets together, preventing their union from being separated into two disjoint non-empty open subsets.

However, if A and B are disjoint, their union may or may not be connected depending on the situation. Therefore, we cannot simply say that the union of any two connected sets is always connected.

Hence, option C is correct.

16. Which of the following is true ?

(a) Closed subset of a compact space is connected

(b) Open subset of a compact space is connected

(c) Closed subset of a compact space is compact

(d) Open subset of a compact space is compact

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Answer: C. Closed subset of a compact space is compact

Explanation:
A fundamental theorem in topology states that every closed subset of a compact space is itself compact.

Suppose X is compact and A is a closed subset of X. Consider any open cover of A. Since A is closed, $X - A$ is open.

Adding $X - A$ to the open cover of A produces an open cover of the entire compact space X.

Since X is compact, this open cover has a finite subcover. Removing $X - A$, if necessary, leaves a finite collection covering A.

Therefore, A is compact.

Closed subsets of compact spaces need not be connected, and open subsets of compact spaces need not necessarily be compact.

Hence, option C is correct.

17. If x and y are orthogonal vector in a Hilbert space H. Then :

$$\text{I: } \|x+y\|^2 = \|x-y\|^2$$
$$\text{II: } \|x-y\|^2 = \|x\|^2 + \|y\|^2$$
$$\text{III: } \|x+y\|^2 = \|x\|^2 + \|y\|^2$$

(a) II and III alone true

(b) II alone true

(c) I, II, III all are true

(d) III alone true

Answer: C. I, II and III are all true

Explanation:
Since x and y are orthogonal,

$$\langle x, y\rangle = 0.$$

Using the Pythagorean theorem in a Hilbert space,

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2.$$

Therefore, statement III is true.

Similarly,

$$\|x-y\|^2 = \|x\|^2 + \|y\|^2$$

because x and $-y$ are also orthogonal.

Therefore, statement II is true.

Since both expressions equal the same quantity,

$$\|x+y\|^2 = \|x-y\|^2.$$

Therefore, statement I is also true.

Thus, all three statements I, II and III are true, making option C correct.

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18. In an inner product space C, u and v belong to an orthonormal set, then

$$\frac{\|U-V\|}{\|U+V\|}$$

is :

(a) 4

(b) 2

(c) 0

(d) 1

Answer: D. 1

Explanation:
Since u and v belong to an orthonormal set,

$$\|u\| = 1,$$

$$\|v\| = 1,$$

and

$$\langle u, v\rangle = 0.$$

Therefore,

$$\|u-v\|^2 = \|u\|^2 + \|v\|^2 = 1+1 = 2.$$

Hence,

$$\|u-v\| = \sqrt{2}.$$

Similarly,

$$\|u+v\|^2 = \|u\|^2 + \|v\|^2 = 2.$$

Therefore,

$$\|u+v\| = \sqrt{2}.$$

Thus,

$$\frac{\|u-v\|}{\|u+v\|} = \frac{\sqrt{2}}{\sqrt{2}} = 1.$$

Hence, option D is correct.

19.

I. Cantor set is of measure zero
II. Cantor set is a nowhere dense set
III. Cantor set is a countable set
IV. Cantor set is a perfect set

(a) All I, II, III and IV are true

(b) I, II and IV are correct

(c) I and II are correct and III and IV are not correct

(d) I and III alone correct

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Answer: B. I, II and IV are correct

Explanation:
The Cantor set has several important properties.

Statement I is true. The Cantor set has Lebesgue measure zero. Although it contains infinitely many points, the total length removed during its construction is 1, leaving a set of measure zero.

Statement II is true. The Cantor set is nowhere dense because its closure has empty interior. In fact, the Cantor set itself is closed, but it contains no non-empty open interval.

Statement III is false. The Cantor set is not countable. It is uncountable and, in fact, has the same cardinality as the set of real numbers.

Statement IV is true. The Cantor set is perfect because it is closed and every point of the Cantor set is a limit point of the set. It has no isolated points.

Therefore, statements I, II and IV are correct.

Hence, option B is correct.

20. The value of

$$\int_1^{\infty} \frac{dx}{x}$$

is :

(a) 0

(b) 1

(c) $\log x$

(d) ∞

Answer: D. ∞

Explanation:
The given improper integral is

$$\int_1^{\infty} \frac{dx}{x}.$$

To evaluate it, introduce an upper limit B:

$$\int_1^{B} \frac{dx}{x}.$$

Since

$$\int \frac{dx}{x} = \ln x,$$

we get

$$\int_1^{B} \frac{dx}{x} = \ln B - \ln 1 = \ln B.$$

Now take the limit as B approaches infinity:

$$\lim_{B \to \infty} \ln B = \infty.$$

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Therefore,

$$\int_1^{\infty} \frac{dx}{x} = \infty.$$

This means that the improper integral diverges.

This integral is closely related to the harmonic series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

which also diverges.

Hence, option D is correct.

21. The characteristic equations of

$$u_{xx} + 2u_{xy} + \sin^2(x)u_{yy} + u_y = 0,$$

when it is of hyperbolic type, are :

(a) $y + x - \cos x$ and $y + x + \cos x$

(b) $y - x + \sin x$ and $y - x - \sin x$

(c) $y - x + \cos x$ and $y + x - \cos x$

(d) $y + \sin x - \cos x$ and $y + \sin x + \cos x$

Answer: B. $y - x + \sin x$ and $y - x - \sin x$

Explanation:

The given second-order PDE is

$$u_{xx} + 2u_{xy} + \sin^2 x\, u_{yy} + u_y = 0.$$

For a second-order PDE of the form

$$Au_{xx} + 2Bu_{xy} + Cu_{yy} + \text{lower-order terms} = 0,$$

we have

$$A = 1, \quad B = 1, \quad C = \sin^2 x.$$

The equation is hyperbolic when

$$B^2 - AC > 0.$$

Therefore,

$$1 - \sin^2 x > 0$$

$$\cos^2 x > 0.$$

Thus, it is hyperbolic wherever $\cos x \neq 0$.

The characteristic equation is

$$A(dy)^2 - 2B\, dx\, dy + C(dx)^2 = 0.$$

Therefore,

$$\left(\frac{dy}{dx}\right)^2 - 2\left(\frac{dy}{dx}\right) + \sin^2 x = 0.$$

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Solving,

$$\frac{dy}{dx} = 1 \pm \sqrt{1 - \sin^2 x}$$
$$= 1 \pm \cos x.$$

Integrating the two equations gives

$$y = x + \sin x + C_1$$

and

$$y = x - \sin x + C_2.$$

Therefore, the two characteristic families can be written as

$$y - x - \sin x = \text{constant}$$

and

$$y - x + \sin x = \text{constant}.$$

Hence, option B is correct.

22. The wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

is :

(a) Parabolic and homogeneous

(b) Parabolic and non-homogeneous

(c) Hyperbolic and homogeneous

(d) Hyperbolic and non-homogeneous

Answer: C. Hyperbolic and homogeneous

Explanation:
The wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u.$$

Rearranging,

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0.$$

The wave equation is a standard example of a hyperbolic partial differential equation. Hyperbolic equations generally describe wave propagation and disturbances travelling with finite speed.

It is also homogeneous because the right-hand side after rearrangement is zero. There is no independent forcing or source term.

For example, in one spatial dimension,

$$u_{tt} - c^2 u_{xx} = 0$$

is the standard one-dimensional homogeneous wave equation.

Therefore, option C is correct.

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23. Which of the following is not Exact?

(a) $2xy\,dx + (x^2 + 3y^2)\,dy = 0$

(b) $(x^2 + xy)\,dx + xy\,dy = 0$

(c) $e^x\,dx + [e^y(y+1)]\,dy = 0$

(d) $\cos x \cos^2 y\,dx - \sin x \sin 2y\,dy = 0$

Answer: B. $(x^2 + xy)\,dx + xy\,dy = 0$

Explanation:

A differential equation

$$M\,dx + N\,dy = 0$$

is exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

For (a),

$$M = 2xy,$$
$$N = x^2 + 3y^2.$$

Then,

$$\frac{\partial M}{\partial y} = 2x$$

and

$$\frac{\partial N}{\partial x} = 2x.$$

Therefore, (a) is exact.

For (b),

$$M = x^2 + xy,$$
$$N = xy.$$

Then,

$$\frac{\partial M}{\partial y} = x,$$

while

$$\frac{\partial N}{\partial x} = y.$$

Since x and y are not generally equal,

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}.$$

Therefore, (b) is not exact.

For (c),

$$M = e^x,$$
$$N = e^y(y+1).$$

Here,

$$\frac{\partial M}{\partial y} = 0$$

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and

$$\frac{\partial N}{\partial x} = 0.$$

Therefore, it is exact.

For (d),

$$M = \cos x \cos^2 y,$$

$$N = -\sin x \sin 2y.$$

Now,

$$\frac{\partial M}{\partial y} = -\cos x \sin 2y$$

and

$$\frac{\partial N}{\partial x} = -\cos x \sin 2y.$$

Therefore, it is exact.

Hence, only (b) is not exact, so option B is correct.

24. Let

$$\phi_1(x) = e^{r_1 x}, \quad \phi_2(x) = xe^{r_1 x}, \quad \phi_3(x) = x^2 e^{r_1 x}.$$

Then the Wronskian $W(\phi_1, \phi_2, \phi_3)$ at $x = 0$ is :

(a) -1

(b) 1

(c) 2

(d) 0

Answer: C. 2

Explanation:

Given

$$\phi_1(x) = e^{r_1 x},$$

$$\phi_2(x) = xe^{r_1 x},$$

$$\phi_3(x) = x^2 e^{r_1 x}.$$

The Wronskian is

$$W = \begin{vmatrix} \phi_1 & \phi_2 & \phi_3 \\ \phi_1' & \phi_2' & \phi_3' \\ \phi_1'' & \phi_2'' & \phi_3'' \end{vmatrix}.$$

At $x = 0$,

$$\phi_1(0) = 1,$$

$$\phi_2(0) = 0,$$

$$\phi_3(0) = 0.$$

For the first derivatives,

$$\phi_1'(0) = r_1,$$

$$\phi_2'(0) = 1,$$

$$\phi_3'(0) = 0.$$

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For the second derivatives,

$$\phi_1''(0) = r_1^2,$$
$$\phi_2''(0) = 2r_1,$$
$$\phi_3''(0) = 2.$$

Therefore,

$$W(0) = \begin{vmatrix} 1 & 0 & 0 \\ r_1 & 1 & 0 \\ r_1^2 & 2r_1 & 2 \end{vmatrix}.$$

This is a lower triangular matrix, so its determinant is the product of its diagonal entries:

$$W(0) = 1 \times 1 \times 2 = 2.$$

Therefore, option C is correct.

25. Consider the equation of constraint

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 - l^2 = 0$$

of the motion in the xy plane of two particles with coordinates (x_1, y_1) and (x_2, y_2) connected by a rigid rod of constant length l. Then the system is :

(a) Holonomic, Scleronomic and Bilateral

(b) Non-holonomic, Rhenomic and Unilateral

(c) Holonomic, Rhenomic and Unilateral

(d) Non-holonomic, Scleronomic and Bilateral

Answer: A. Holonomic, Scleronomic and Bilateral

Explanation:
The constraint is

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 - l^2 = 0.$$

This states that the distance between the two particles remains equal to the constant length l of the rigid rod.

It is holonomic because the constraint can be expressed as an equation involving the coordinates of the particles:

$$f(x_1, y_1, x_2, y_2) = 0.$$

It is scleronomic because time t does not appear explicitly in the constraint equation. Thus, the constraint itself does not change with time.

It is bilateral because it is expressed as an equality rather than an inequality. The particles must always remain exactly the fixed distance l apart.

Therefore, the system is holonomic, scleronomic and bilateral.

Hence, option A is correct.

26. For what value of n do the transformation equations

$$Q = \sqrt{q}\cos np, \qquad P = \sqrt{q}\sin np$$

is canonical ?

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(a) $n = 1$

(b) $n = 3$

(c) $n = \dfrac{1}{2}$

(d) $n = 2$

Answer: D. $n = 2$

Explanation:

The transformation is

$$Q = \sqrt{q}\cos(np),$$
$$P = \sqrt{q}\sin(np).$$

For the transformation to be canonical, the Poisson bracket must satisfy

$$\{Q, P\} = 1.$$

Calculate the required derivatives:

$$\frac{\partial Q}{\partial q} = \frac{\cos(np)}{2\sqrt{q}},$$
$$\frac{\partial Q}{\partial p} = -n\sqrt{q}\sin(np),$$
$$\frac{\partial P}{\partial q} = \frac{\sin(np)}{2\sqrt{q}},$$
$$\frac{\partial P}{\partial p} = n\sqrt{q}\cos(np).$$

Therefore,

$$\{Q, P\} = \left(\frac{\partial Q}{\partial q}\right)\left(\frac{\partial P}{\partial p}\right) - \left(\frac{\partial Q}{\partial p}\right)\left(\frac{\partial P}{\partial q}\right)$$
$$= \frac{n}{2}\cos^2(np) + \frac{n}{2}\sin^2(np).$$

Using

$$\sin^2(np) + \cos^2(np) = 1,$$

we get

$$\{Q, P\} = \frac{n}{2}.$$

For a canonical transformation,

$$\frac{n}{2} = 1.$$

Therefore,

$$n = 2.$$

Hence, option D is correct.

27. For the velocity field

$$V_1 = kx_2, \qquad V_2 = V_3 = 0,$$

the rate of deformation is :

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(a)

$$\begin{bmatrix} 0 & \frac{k}{2} & 0 \\ -\frac{k}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0 & k & 0 \\ -k & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 & k & 0 \\ k & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & \frac{k}{2} & 0 \\ \frac{k}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer: D

Explanation:
The velocity field is

$$V_1 = kx_2,$$
$$V_2 = 0,$$
$$V_3 = 0.$$

The rate-of-deformation tensor is the symmetric part of the velocity-gradient tensor:

$$D_{ij} = \frac{1}{2}\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right).$$

Here,

$$\frac{\partial V_1}{\partial x_2} = k.$$

All other relevant velocity derivatives are zero.

Therefore,

$$D_{12} = D_{21} = \frac{k}{2}.$$

The diagonal components are zero, and the components involving the third direction are also zero.

Hence, the rate-of-deformation tensor is

$$\begin{bmatrix} 0 & \frac{k}{2} & 0 \\ \frac{k}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

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Notice that a rate-of-deformation tensor must be symmetric. This is also why options containing $k/2$ and $-k/2$ in opposite positions cannot represent the deformation tensor.

Therefore, option D is correct.

28. For the velocity field

$$V_1 = \frac{x_1}{1+t}, \qquad V_2 = \frac{x_2}{1+t}, \qquad V_3 = \frac{x_3}{1+t}$$

the density ρ of a material particle as a function of time is :

where ρ_0 is the density at the initial time $t_0 = 0$.

(a) $\rho = \dfrac{\rho_0}{(1+t)^3}$

(b) $\rho = \dfrac{\rho_0}{(1-t)^3}$

(c) $\rho = \dfrac{\rho_0}{(1+t)^2}$

(d) $\rho = \dfrac{\rho_0}{(1-t)^2}$

Answer: A. $\rho = \dfrac{\rho_0}{(1+t)^3}$, assuming the initial time $t_0 = 0$

Explanation:
The velocity field is

$$V_1 = \frac{x_1}{1+t},$$
$$V_2 = \frac{x_2}{1+t},$$
$$V_3 = \frac{x_3}{1+t}.$$

For a compressible fluid, conservation of mass gives the continuity equation

$$\frac{D\rho}{Dt} + \rho \operatorname{div} V = 0.$$

Calculate the divergence:

$$\operatorname{div} V = \frac{\partial V_1}{\partial x_1} + \frac{\partial V_2}{\partial x_2} + \frac{\partial V_3}{\partial x_3}.$$

Therefore,

$$\operatorname{div} V = \frac{1}{1+t} + \frac{1}{1+t} + \frac{1}{1+t} = \frac{3}{1+t}.$$

Hence,

$$\frac{d\rho}{dt} + \frac{3\rho}{1+t} = 0.$$

Rearranging,

$$\frac{d\rho}{\rho} = -\frac{3\,dt}{1+t}.$$

Integrating,

$$\ln \rho = -3\ln(1+t) + \text{constant}.$$

Therefore,

$$\rho = \frac{C}{(1+t)^3}.$$

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If $\rho = \rho_0$ at the initial time $t = 0$, then $C = \rho_0$.

Thus,

$$\rho = \frac{\rho_0}{(1+t)^3}.$$

Hence, option A is correct.

Strictly, if the initial time is an arbitrary t_0 rather than zero, the more general result is

$$\rho = \rho_0 \left[\frac{1+t_0}{1+t}\right]^3.$$

29. Choose the correct formula for back substitution in direct methods.

(a)

$$x_1 = \frac{b_1 - \sum_{j=2}^{n} a_{ij}x_j}{a_{11}}$$

(b)

$$x_n = \frac{b_n - \sum_{j=1}^{n-1} a_{nj}x_j}{a_{nn}}$$

(c)

$$x_1 = \frac{b_j}{a_{ii}}, \qquad i = 1(1)n, \quad a_n \neq 0, \quad i = 1(1)n$$

(d)

$$x_1 = \frac{1}{|A|}\text{adj}(A)b = A^{-1}b$$

Answer: A

Explanation:

In direct methods such as Gaussian elimination, the original system is first reduced to an upper triangular system.

The final equation gives x_n first:

$$x_n = \frac{b_n}{a_{nn}}.$$

Then the remaining unknowns are calculated successively in reverse order.

The general back-substitution formula is

$$x_i = \frac{b_i - \sum_{j=i+1}^{n} a_{ij}x_j}{a_{ii}}.$$

For the final step, when $i = 1$,

$$x_1 = \frac{b_1 - \sum_{j=2}^{n} a_{1j}x_j}{a_{11}}.$$

This corresponds to the expression intended in option A.

Therefore, option A is correct.

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30. The truncation error of the Lagrange quadratic interpolating polynomial is :

(a)
$$E_2(f;x) = f(x) - P_2(x)$$

(b)
$$P_2(x) = a_0 + a_1 x + a_2 x^2$$

(c)
$$|f''(x)| \leq M_2, \qquad x \in [x_0, x_1]$$

(d)
$$|f(x) - P(x)| \leq \frac{1}{8}(x_1 - x_0)^2 M_2$$

Answer: A. $E_2(f;x) = f(x) - P_2(x)$

Explanation:
A quadratic Lagrange interpolation polynomial $P_2(x)$ approximates a function $f(x)$ using three interpolation points.

The truncation error is defined as the difference between the actual function value and the interpolated value:
$$E_2(f;x) = f(x) - P_2(x).$$

Therefore, option A correctly defines the truncation error.

More specifically, if the interpolation points are x_0, x_1 and x_2, then the Lagrange interpolation error can be written as
$$E_2(f;x) = \frac{f'''(\xi)}{3!}(x - x_0)(x - x_1)(x - x_2),$$

for some ξ lying in the relevant interval.

Thus, a quadratic interpolation polynomial involves the third derivative in its detailed error formula.

Option B merely gives the general form of a quadratic polynomial and does not represent its truncation error. Options C and D are associated with different error estimates and do not give the general quadratic Lagrange truncation error.

Therefore, option A is correct.

31. The deterioration of an approximation as the order of the derivative increases is associated with:

(a) Nodal points

(b) Uniform Nodal points

(c) Non - uniform nodal points

(d) Error of approximations

Answer: D. Error of approximations

Explanation:
In numerical approximation, a function may be approximated quite accurately, but when we

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differentiate the approximating function repeatedly, the accuracy can deteriorate. Small errors already present in the approximation may become magnified when derivatives of higher order are calculated.

This deterioration becomes more significant as the order of differentiation increases. Such behaviour is associated with the error in numerical approximation rather than with the nodal points themselves.

Nodal points are simply the points at which function values are specified for interpolation or approximation. They may be uniformly or non-uniformly distributed, but these terms do not describe the deterioration of accuracy.

Therefore, among the given options, D. Error of approximations is the appropriate answer

32. If the cumulative effect of all errors, including round - off errors is bounded, independent of the number of mesh points is called

(a) Convergent

(b) Single step method

(c) Multistep method

(d) Stable

Answer: D. Stable

Explanation:
A numerical method is said to be stable when small errors introduced during computation do not grow without bound as the calculation proceeds.

During numerical computation, several types of errors can occur, including round-off errors, truncation errors and errors in the initial data.

If the cumulative effect of these errors remains bounded independently of the number of mesh points or computational steps, the numerical procedure is called stable.

Stability is particularly important because computers always perform calculations with finite precision. An unstable method can magnify even a very small round-off error and eventually produce a completely inaccurate answer.

Therefore, option D is correct.

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33. Let $T(G)$ denote the number of spanning trees of G, and $K_{m,n}$ denote the complete bipartite graph with m and n vertices in a Biparties graph, then $T(K_{2,n}) =$ ____ .

(a) $n2^{n-1}$

(b) $n2^{n+1}$

(c) $(n-1)2^{n-1}$

(d) $(n+1)2^{n+1}$

Answer: A. $n2^{n-1}$

Explanation:
For a complete bipartite graph $K_{m,n}$, the number of spanning trees is given by the standard formula

$$T(K_{m,n}) = m^{n-1}n^{m-1}.$$

Here, the graph is $K_{2,n}$.

Therefore, put $m = 2$:

$$T(K_{2,n}) = 2^{n-1}n^{2-1} = 2^{n-1}n.$$

Thus,

$$T(K_{2,n}) = n2^{n-1}.$$

For example, if $n = 2$, then $K_{2,2}$ has

$$2 \times 2^1 = 4$$

spanning trees.

Hence, option A is correct.

34. A k-critical graph is k-chromatic. Then the number of vertices of the graph satisfies:

(a) $|V(G)| = k$

(b) $|V(G)| \geq k$

(c) $|V(G)| = k + 1$

(d) $|V(G)| = k + 2$

Answer:B. $|V(G)| \geq k$

Explanation: A k-critical graph is a graph whose chromatic number is k, but every proper subgraph has chromatic number less than k.

A graph requiring k colours must contain at least k vertices. Hence,

$$|V(G)| \geq k.$$

For example, the complete graph K_k is k-critical and has exactly k vertices, while other k-critical graphs may contain more than k vertices.

Therefore, the correct answer is

$$|V(G)| \geq k.$$

35. Let G be a simple graph with $2n + 1$ vertices and maximum degree Δ. If the number of edges of G is greater than $n\Delta$, then the edge chromatic number $\chi'(G)$ is equal to:

(a) Δ

(b) $\Delta + 1$

(c) $\Delta - 1$

(d) $\Delta + 2$

Answer: B. $\Delta + 1$

Explanation:

Since G has $2n + 1$ vertices, any matching in G can contain at most n edges.

In a proper edge colouring, all edges having the same colour form a matching. Therefore, if only Δ colours are used, the maximum possible number of edges is

$$n\Delta.$$

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But the given condition is

$$|E(G)| > n\Delta.$$

Hence,

$$\chi'(G) > \Delta.$$

By Vizing's theorem, for every simple graph,

$$\Delta \leq \chi'(G) \leq \Delta + 1.$$

Therefore,

$$\chi'(G) = \Delta + 1.$$

Hence, option B is correct.

36. Let G be a simple bipartite graph with ε number of edges and γ number of vertices, then :

(a) $\varepsilon > \dfrac{\gamma^2}{4}$

(b) $\varepsilon \leq \dfrac{\gamma^2}{4}$

(c) $\varepsilon > \dfrac{(\gamma+1)^2}{4}$

(d) $\varepsilon \geq \dfrac{(\gamma+1)^2}{4}$

Answer: B. $\varepsilon \leq \dfrac{\gamma^2}{4}$

Explanation:

Let G be a simple bipartite graph having γ vertices and ε edges.

Suppose the two parts of the bipartition contain m and n vertices. Then

$$m + n = \gamma.$$

Because the graph is simple and bipartite, the maximum possible number of edges occurs when every vertex in one part is connected to every vertex in the other part.

Thus,

$$\varepsilon \leq mn.$$

For a fixed sum $m + n = \gamma$, the product mn is maximum when m and n are as nearly equal as possible.

Using

$$mn \leq \frac{(m+n)^2}{4},$$

we obtain

$$mn \leq \frac{\gamma^2}{4}.$$

Therefore,

$$\varepsilon \leq \frac{\gamma^2}{4}.$$

This is essentially the extremal bound achieved by a balanced complete bipartite graph.

Hence, option B is correct.

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37. An animal feed company produce 200kg of mixture consisting of ingredients x_1 and x_2. x_1 costs Rs.3 per kg and x_2 Rs.8 per kg. no more that 80kg of x_1 can be used and atleast 60kg of x_2 must be used. The LP model to optimize the cost is:

(a)

$$\begin{aligned}&\text{Minimize } z = 3x_1 + 8x_2\\&\text{Subject to } x_1 + x_2 = 200\\&x_1 \leq 80\\&x_2 \geq 60\\&x_1, x_2 \geq 0\end{aligned}$$

(b)

$$\begin{aligned}&\text{Maximize } z = 3x_1 + 8x_2\\&\text{Subject to } x_1 + x_2 = 200\\&x_1 \leq 80\\&x_2 \geq 60\\&x_1, x_2 \geq 0\end{aligned}$$

(c)

$$\begin{aligned}&\text{Minimize } z = 3x_1 + 8x_2\\&\text{Subject to } x_1 + x_2 = 200\\&x_1 \geq 80\\&x_2 \leq 60\\&x_1, x_2 \geq 0\end{aligned}$$

(d)

$$\begin{aligned}&\text{Minimize } z = 3x_1 + 8x_2\\&\text{Subject to } x_1 + x_2 = 200\\&x_1 \geq 80\\&x_2 \geq 60\\&x_1, x_2 \geq 0\end{aligned}$$

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Answer: A

Explanation:
Let

$$\begin{aligned}x_1 &= \text{kilograms of ingredient 1,}\\x_2 &= \text{kilograms of ingredient 2.}\end{aligned}$$

Ingredient x_1 costs Rs. 3 per kg, while ingredient x_2 costs Rs. 8 per kg.

Therefore, the total cost is

$$Z = 3x_1 + 8x_2.$$

Since the company wants to optimize the cost of producing the mixture, the natural objective is to minimize the total cost.

Therefore,

$$\text{Minimize } Z = 3x_1 + 8x_2.$$

The total mixture must weigh exactly 200 kg. Hence,

$$x_1 + x_2 = 200.$$

The question states that no more than 80 kg of x_1 can be used. "No more than" means less than or equal to:

$$x_1 \leq 80.$$

At least 60 kg of x_2 must be used. "At least" means greater than or equal to:

$$x_2 \geq 60.$$

Finally, quantities cannot be negative:

$$x_1 \geq 0 \text{ and } x_2 \geq 0.$$

Therefore, the complete LP model is

$$\text{Minimize } Z = 3x_1 + 8x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &= 200, \\ x_1 &\leq 80, \\ x_2 &\geq 60, \\ x_1, x_2 &\geq 0. \end{aligned}$$

Hence, option A is correct.

38. If each cost is reduced by Rs. 10 in 3 jobs, 5 machines assignment problem, the optimal cost is :

(a) reduced by 15

(b) reduced by 9

(c) reduced by 25

(d) reduced by 30

Answer: D. Reduced by Rs. 30

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Explanation:

This is an assignment problem involving 3 jobs and 5 machines.

Although there are 5 machines, only 3 assignments are ultimately made because there are only 3 jobs. Each job must be assigned to one machine.

Suppose every cost element in the assignment matrix is reduced by Rs. 10.

Each selected assignment therefore costs Rs. 10 less than before.

Since the optimal solution contains 3 actual job-machine assignments, the total reduction is

$$3 \times 10 = \text{Rs. } 30.$$

The identity of the optimal assignment may remain unchanged because the same constant has been subtracted from every relevant cost.

Therefore, the optimal total cost is reduced by Rs. 30.

Hence, option D is correct.

39. Principal stress values of the stress tensor

$$\sigma_{ij} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

is:

(a) $2, 1, 4$

(b) $-2, 1, 4$

(c) $2, -1, 4$

(d) $-2, 1, -4$

Answer: B. $-2, 1, 4$

Explanation:
The principal stresses are the eigenvalues of the stress tensor

$$\sigma = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}.$$

To obtain the principal stresses, solve the characteristic equation

$$\det(\sigma - \lambda I) = 0.$$

Thus,

$$\begin{vmatrix} 3-\lambda & 1 & 1 \\ 1 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{vmatrix} = 0.$$

Expanding the determinant gives the characteristic polynomial

$$\lambda^3 - 3\lambda^2 - 6\lambda + 8 = 0.$$

This can be factorized as

$$(\lambda - 1)(\lambda - 4)(\lambda + 2) = 0.$$

Therefore,

$$\lambda = 1,$$
$$\lambda = 4,$$
$$\lambda = -2.$$

Hence, the three principal stress values are

$$-2,\ 1 \text{ and } 4.$$

Therefore, option B is correct.

40. If

$$\frac{p}{q} + \frac{1}{2}\left[v + \frac{2\mu(a^2 - x^2)}{(a^2 + x^2)^2}\right]^2 = \text{constant}, \qquad q > 0,$$

then p is maximum when:

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(a) $\dfrac{2\mu(a^2-x^2)}{(a^2+x^2)^2}$ is maximum

(b) $\dfrac{2\mu(a^2-x^2)}{(a^2+x^2)^2}$ is minimum

(c) $v(a^2+x^2)^2$ is maximum

(d) $\left|v+\dfrac{2\mu(a^2-x^2)}{(a^2+x^2)^2}\right|$ is minimum

Answer: D. $\left|v+\dfrac{2\mu(a^2-x^2)}{(a^2+x^2)^2}\right|$ is minimum

Explanation: Let

$$V = v + \frac{2\mu(a^2-x^2)}{(a^2+x^2)^2}.$$

Then the given equation becomes

$$\frac{p}{q} + \frac{V^2}{2} = C,$$

where C is a constant.

Therefore,

$$p = q\left(C - \frac{V^2}{2}\right).$$

Since $q > 0$, p is maximum when V^2 is minimum.

This occurs when

$$|V|$$

is minimum.

Hence,

$$\left|v+\frac{2\mu(a^2-x^2)}{(a^2+x^2)^2}\right|$$

must be minimum.

Therefore, option D is correct.

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